The Genesis of the Moore Method

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Abstract: R. L. Moore (1882-1974) was one of the towering figures in American mathematics. This paper investigates the genesis of the special teaching method he adopted, now called the Moore Method. We examine the nine-year period that Moore spent at the University of Pennsylvania with a view toward singling out the essential ingredients in the method that he later perfected at the University of Texas. Moore's enduring influence on a long list of distinguished mathematicians can be seen in the careers of his Penn colleague H. H. Mitchell, of his doctoral students J. R. Kline, G. H. Hallett, and A. M. Mullikin, and of some of Kline's doctoral students.

The Moore Method, named after the eminent topologist Robert Lee Moore (1884-1972), is perhaps the most well-known process in the world for training research mathematicians. In spite of its notoriety, however, there appear to be major misconceptions regarding its origin and development. For example, the editor of an MAA publication wrote, "R. L. Moore developed his approach to discovery learning from 1920 to 1969 at the University of Texas." [**21**, p. 6] On the contrary, we supply overwhelming evidence to support the contention that the pivotal period of development occurred between 1911 and 1920, when Moore was in Philadelphia and not in Austin. Overall we trace the evolution of the Moore Method through three distinct periods:

1. Intuition (up to 1911)
2. Development (1911-1920)
3. Maturity (after 1920)

The first period includes Moore's education in high school, college, and graduate school, as well as his first three academic appointments. During this entire time certain elements of the Moore Method were interwoven intuitively into the fabric of Moore's teaching, but conditions were unfavorable for them to coalesce into a cohesive program that could be put into regular practice. The second period took place at the University of Pennsylvania (Penn), from his appointment in 1911 until he left for the University of Texas in 1920. As we will see, Penn provided Moore with ripe conditions for cultivating his method into a finished masterpiece. The final period begins with Moore's first dozen years at Texas, showing how the method was transported to yet another supportive environment, and extends into the early 1950s in order to see how Moore's roots remained planted firmly in Philadelphia.

Our primary focus here is on the Moore Method rather than R. L. Moore's persona or his numerous other achievements. Therefore we only discuss those elements of his life and work that played a definitive role in his teaching. A very informative book by Reginald Traylor [**22**] contains many details about Moore's life, while a paper by the legendary Texan's first Ph.D. student at Texas, R. L. Wilder, provides a comprehensive analysis of his publications. [**26**]

Intuition

Robert Lee Moore was born and raised in Dallas, where his parents operated a hardware store and feed company in the downtown area of the city, just off the town square. The fifth of six children, Robert Lee continued the family tradition of receiving an early education in a private school. From age eight to fifteen he attended a school that existed under different names but one headmaster, Waldewar Malcolmson. Moore obtained an excellent foundation in English, history, and mathematics, but some time before the age of 15 he got the notion to attend the University of Texas. Although the relatively new university had opened its doors only in 1883, by 1898 its entrance requirements included knowledge of Greek or Latin, so Moore withdrew from Malcolmson's school to teach himself Latin. During that year he also set about reading calculus because it was not offered at school, and he enjoyed learning mathematics so much that he wanted to extend his studies.

Moore's manner of learning mathematics – one that he apparently came upon independently – was to read a theorem, cover its proof, attempt to prove it himself, and peruse the proof only when absolutely necessary. The policy that the learner was solely responsible for mastering new material – without aid of any written or verbal sources – would form a cornerstone of the Moore Method.

Even though Moore was not yet 16, the next year he passed all four entrance examinations (in English, history, mathematics, and Latin) and matriculated at the University of Texas in the fall of 1898. He showed his mettle in the classroom at once, and within the first few weeks of the semester (the Texas academic year consisted of three terms) his calculus professor, G. B. Halsted, yanked him out of class and placed him in his junior-level course on projective geometry. The policy of recognizing, and then recruiting, students with talent and promise from calculus and directing their further studies in mathematics became another fundamental element of the Moore Method. Thus it appears that Moore was competing with juniors and seniors after being in college less than a month.

Not surprisingly, R. L. Moore thrived under Halsted's tutelage, graduating in only three years. Indeed, in 1901 he earned both a bachelor's degree and a master's degree. Then he remained at Texas for another year to continue his studies. Extending work with a mentor beyond the completion of formal graduation requirements also became part of the Moore Method, albeit an external one.

A notable event from 1901-1902 sealed Moore's academic fate. During that year Halsted was busy preparing a report for the journal *Science* on a fundamental work by the eminent German mathematician David Hilbert. In response to a letter Halsted sent him, Hilbert suggested an improved wording of one of his postulates, numbered II 4, which sparked Halsted to ask Moore whether that assumption could be proved from the other four assumptions. Moore was able to prove the redundancy at once, and he outlined his approach to Halsted. The latter was so impressed with Moore's proof that he wrote a short note that appeared in the April 1902 issue of the *Monthly*. It began, "In his 'Vorlesung ueber Euklidische Geometrie' … Hilbert gave the most remarkable set of axioms ever created for the founding of geometry." [**3**, p. 98] Halsted continued, "A year ago, while preparing my Report, I discussed with my pupil, R. L. Moore, whether II 4 might not be demonstrated from the other assumptions … Early next morning he announced to me that he had demonstrated Hilbert's new axiom, thus reducing the Betweenness Assumptions from five to four. … [T]he demonstration, as I have written it out from his oral communication, seems of most unexpected simplicity and elegance." Thus although Halsted is listed as the sole author of the paper, he gives full credit to his protégé for the proof. Nonetheless, some sources include the paper on Moore's list of publications (for example, [**1**, p. 243] and [**22**, p. 197]).

In the next issue of the *Monthly*, E. H. Moore published the text of a letter he had sent to the 19-year old R. L. Moore. In it, the famed chair of the mathematics department at the University of Chicago noted that R. L.'s conclusion – that one of Hilbert's axioms was redundant – had already appeared in an article he had just published in the *Transactions*. E. H. acknowledged, however, that R. L. had proceeded independently. Moreover, he praised the teenager for "the delightfully simple proof of the redundancy." [**9**, p. 153]

Moore's success on the international stage emboldened Halsted to campaign for a position as Mathematics Tutor for his promising student for 1902-1903. When the president of the university chose another candidate instead, Halsted included a scathing criticism of the university's regents in an article he published in *Science*. Halsted was fired within six weeks of the journal's publication, even though he had served as chair of the mathematics department since 1884. Nonetheless, he continued his active support for his budding prodigy, just as he had done to support the careers of Henry Fine, when he was a student at Princeton, and Leonard E. Dickson, when he was a student at Texas. Moore himself was relegated to spending the year 1902-1903 as a high school teacher in Marshall, Texas. He was then admitted to the graduate program at the University of Chicago, which was regarded as the best in the country at the time.

Chicago was the perfect fit for R. L. Moore's interests and ability. The emphasis in E. H. Moore's department was on the very highest standards of research. Besides, both Moores' primary interest at the time centered on the axiomatic foundations of geometry. One of E. H. Moore's earliest Ph.D. students, Oswald Veblen, had just completed his dissertation when R. L. Moore appeared on campus for the fall term, 1903. However, Veblen remained at the University of Chicago another two years, during which time he developed a symbiotic relationship with Moore. When Veblen submitted his results the next spring to the *Transactions*, he acknowledged support from his new friend: "I wish to express deep gratitude to Professor E. H. Moore … and also to Messrs. N. J. Lennes and R. L. Moore, who have critically read parts of the manuscript." [**23**, p. 344] Conversely, R. L. Moore benefited from Veblen. When he finally submitted his own 1905 dissertation in December, 1907, he wrote, "I wish to thank Professor E. H. Moore and Professor O. Veblen for suggestions and criticisms. Professor Veblen, who suggested the undertaking of this investigation, has not only made numerous suggestions and criticisms, but has given me much help in the way of actual collaboration." [**10**, p. 488]

Although the 22-year old Moore embarked on an academic career at once, he did not find a suitably accommodating environment for another six years. The preceptorship he held at Princeton from 1906 to 1908 might seem to have offered optimal conditions, especially since Veblen had joined the department the preceding year, but Moore became very upset with the manner of teaching there. He did approve of the smaller sections, whose size was similar to his own undergraduate experience in Texas, and this too became part and parcel of the Moore

Method. However, he was especially agitated with common exams. One notable aspect of the Moore Method is that the instructor moves with deliberate speed, letting the students' progress dictate the rate of material presented, not some externally prepared syllabus. Fortunately Moore found an entirely different atmosphere when he accepted an instructorship at the University of Pennsylvania in 1911.

Development

R. L. Moore remained at Penn for nine fruitful years, arriving in 1911 with a pedigree degree and outstanding promise but a shallow record, and departing in 1920 as a recognized scholar. While he had published but two articles (one of which was his dissertation) in the six prior years at the University of Tennessee, Princeton, and Northwestern, he would pen 17 more during his tenure in Philadelphia. His 1912 paper [**11**] garnered national repute and led to his appointment as an associate editor of the *Transactions* two years later. Not surprisingly, he carried out his editorial duties diligently until he resigned at the end of 1927. A 1916 paper [**12**] on the foundations of plane analysis situs, in which Moore listed the first of several characterizations of the plane, established his reputation internationally. To emphasize the importance of this paper, the main results were summarized in the *Proceedings of the National Academy of Sciences* later that year. [**13**] His first full publication in this prestigious journal appeared two years later. [**14**] The pair of 1916 papers led to his promotion that year to assistant professor. Since our focus here is on the development of the Moore Method of teaching *per se* we present no further discussion of his publication record, directing the interested reader to [**26**] instead.

During the academic year 1912-1913 Moore taught a graduate course titled Foundations of Mathematics. Its description in the Penn catalog reads:

> The theory of positive integers as a basis for analysis. Rigid motion and correspondence with a number manifold as factors in determining the properties of space. Metrical and non-metrical spaces. A critical study of inter-relations between different systems of axioms.

In 1913-1914 Moore instituted a new course called Theory of Point Sets, described as follows in that year's Penn catalog:

> Theory of sets of points in metrical and in non-metrical spaces. Contributions of Fréchet and others to the foundations of point set theory. Content and measure. Jordan curve theory and other applications.

These two courses formed the cornerstone of a longer list of courses that came to comprise the Moore Method after Moore departed for Texas. (It might be noted in passing here that the course on point set theory was offered slightly before the publication of Hausdorff's landmark book on topology, *Grundzüge der Mengenlehre*.)

It is well known that R. L. Moore exerted a profound influence on the students who worked under him but it is not so well known that he also had a considerable affect on the teaching methods of some of his colleagues. In a 1971 interview with R. Traylor (see [**22**, p. 85] for a fuller account) George Hallett, who became Moore's second doctoral student in 1918, described a course taught by H. H. Mitchell (1885-1943), who, like Moore, had come to Penn as an instructor in 1911. Hallett recalled:

> Professor Mitchell … gave us this book, but asked us to go through all the proofs that were given and find out whether they were watertight proofs or not and if not, why not. … Actually this course had many elements of the other course. We'd discuss the matters in class together.

Hallett's account recalls a course taught by H. H. Mitchell using a real variables textbook written by Yale's James Pierpont's [**20**]. Reference in the penultimate sentence to "the other course" probably points to Moore's course on the foundations of mathematics since, according to Penn's graduate catalog, he offered it during Hallett's first year of matriculation. Since Hallett took courses from 1916 to 1918 we thus find that by 1918 other mathematics professors were adapting Moore's manner of teaching to accommodate their own personalities and circumstances. This undoubtedly represents the first example of a course taught in what is now called a modified Moore Method. Consequently, titles of sessions like one conducted at the annual AMS-MAA meeting in 2003, "Successful strategies for implementing a Texas-style (modified Moore Method) course", might require modest editing to achieve historical accuracy.

Moore's enduring fame with his teaching method, as opposed to his outstanding accomplishments in point set topology, rests on the quality of his doctoral students. Albert Lewis has called Moore's 50 Ph.D. students "the most distinguished group of mathematicians in the United States to have been taught by the same person" [**7**, p. 652]. Although the vast majority of these graduates were produced in Texas, a review of his activities regarding the three students from Penn provides conclusive evidence that the Moore Method was fully developed by the time Moore left for Texas in 1920.

Moore's first doctoral student was John Robert Kline (1891-1955), a native of the Philadelphia area who graduated as valedictorian of his 1912 class at Muhlenberg College in Allentown. After teaching the next year at Allentown Preparatory School, Kline enrolled at the University of Pennsylvania in the fall of 1913, just in time to take R. L. Moore's initial offering of the Theory of Point Sets. In a later letter, Kline teased Moore about not remembering a result that was proved in that course. He wrote, "In 1914 when I was in your class you built up an example showing [the so-called Theorem of Hobson to be false]." [**A2**] This statement provides clear evidence that Moore was teaching in his inimitable style as early as then, even if he later forgot some of the details of the material he introduced in the course.

It has been said that, "At Pennsylvania, Moore's best student was J. R. Kline, who served for many years as Secretary of the AMS." [**19**, p. 450] Kline achieved the first notable success for the effectiveness of the Moore Method in its earliest manifestation. After paying full tuition for his first year at Penn, he was rewarded for his outstanding record with a Harrison Fellowship for the academic year 1914-1915. He then returned to Allentown to teach at his *alma mater*, Muhlenberg. An article in the June 1, 1915, issue of an Allentown newspaper announcing that day's wedding revealed, "Following the ceremony, Prof. Robert Lee Moore … will tender a luncheon to the newly married couple." [**A1**] Such an action on Moore's part certainly belies the folklore that Moore was vehemently opposed to his students marrying before completing all requirements for the degree. Kline finished his dissertation during his year at Muhlenberg, being awarded the Ph.D. in 1916. He published it in the *Annals of Mathematics* later in the year. [**6**] It might be of some interest to point out that Penn faculty member H. H. Mitchell was one of the editors of the *Annals* at that time.

The awarding of the degree was notable for Moore but it was not the only success of the Moore Method that year. Recall that he had published only two papers before arriving at Penn, one of which was his dissertation. It appears from his correspondence with Veblen during 1905-1906, when Moore was at Tennessee and Veblen at Princeton, that Moore had been working on a project to axiomatize the positive integers without postulating an order relationship. (This discovery seems to have been made first by R. L. Wilder in 1982. See [**26**, pp. 75-76].) Unfortunately there are no extant copies of the paper Moore apparently submitted to the *Annals* on this topic, but Veblen encouraged his friend to drop the matter and turn full attention to preparing his dissertation for publication. Moore did not follow this advice, perhaps due to malaria which he contracted over the summer that he spent in Texas between stints at Tennessee and Princeton. This lingering disease severely limited his activities during his first year in the East. Moore did publish his dissertation in October 1908, but he had submitted it the preceding December, during his second – and final – year at Princeton.

What does this have to do with Kline and the Moore Method? Veblen suggested to Moore that his paper on the axiomization of the positive integers was rejected because *Annals* editor E. V. Huntington, the Harvard postulational theorist, did not fully understand the wording of Moore's deep (and verbose) axioms. (The reader can get an idea of the depth and verbosity of other Moore axioms by examining Axiom 1 in his classical book on topology [**16**, p. 6].) Karen Parshall has recently uncovered evidence of keen competition within the American school of postulate theorists pitting the eastern ensemble (notably Harvard's E. V. Huntington), against the Chicago contingent (including E. H. Moore, Oswald Veblen, L. E. Dickson, and R. L. Moore). She wrote, "Especially during the first five years of the new century, competition was stiff between Huntington and the Chicagoans." [**18**] Our account below will suggest that the rivalry extended through the second decade of the 20th century as well. Therefore at first glance it appears entirely surprising to find a joint paper by Huntington and R. L. Moore's first Ph.D. student as early as 1916. Upon closer inspection, the second author is listed as J. Robert Kline, the only time this form of his name appears in print in any document bearing Kline's name. Fortunately an otherwise routine footnote reveals how the paper evolved into a joint effort. The footnote reads [**5**, p. 301]:

> The parts of the paper which do not involve the postulates here numbered 5 and 8 were presented by Professor Huntington at the meetings of December 31, 1912, and April 26, 1913. The necessity of adding postulates 5 and 8 was kindly pointed out by Professor R. L. Moore, and all the theorems and examples which involve these two postulates are due to Dr. Kline.

Edward Huntington had presented the paper before two AMS audiences several years before presenting it a third time on September 5, 1916. It seems clear from the listing of Kline's name and the wording of the footnote that initially Huntington had written the paper alone but was forced into sharing authorship due to Kline's painstaking analysis. Either R. L. Moore heard the lectures or word of their content reached him indirectly, but the end result is that when Huntington's postulates were subjected to the analytical scrutiny of someone schooled in the Moore Method, especially postulates dealing with the betweenness assumptions (recall the Halsted-Moore affair of 1902), they did not pass muster. Huntington was forced to rewrite the paper and add a coauthor who was a student of a postulational rival. One can only imagine Moore and Kline celebrating their victory over their prestigious foe in the basement of Penn's College Hall, especially in light of Moore's competitive nature. Cast in this light, perhaps the word *kindly* in Huntington's footnote would be better served by *gleefully*.

With these two successes in mind, J. R. Kline returned to Penn for two more years, supported in 1916-1917 by a second Harrison Fellowship for Research in Mathematics and in 1917-1918 as an instructor in the department. It became a trademark of many early Moore graduates to spend a postdoctoral year or two with the master before starting out on their own. As we will see, sometimes Moore's students went elsewhere to spend a postdoctoral period with another member of the emergent Moore family. For Kline the collaboration resulted in the only joint paper Moore ever wrote [**17**], and only the second partnership that included Kline as an author. After his two years with Moore, Kline left Penn for one year at Yale and another at the University of Illinois before returning to Philadelphia.

Moore's second doctoral student, George H. Hallett, Jr. (1895-1985), left Penn the same year as Kline, 1918, without remaining for additional study. Hallett came from excellent mathematical stock – his father was a colleague of Moore's who had obtained his own Penn doctorate in 1896 (without a dissertation supervisor). The younger Hallett spent a lifetime in government work, notably with the Proportional Representation League and the Citizens Union. His 1937 book on proportional representation became a landmark in political theory. [**2**] When he died in 1981 the American Political Science Association, the world's largest professional organization for the study of politics, created the George H. Hallett Award in his honor. The award is presented annually to the author of a book (published at least 10 years earlier) that made important contributions to the literature on representation and electoral systems.

In a 1971 interview with Reginald Traylor, Dr. Hallett acknowledged his debt to the special training he had received at Penn even though more than 50 years had elapsed since he had completed his graduate studies under R. L. Moore. Hallett recalled [**22**, p. 86]:

> Dr. Moore's method of teaching brought out what appeared to me to be the two most important faculties in mathematical research … imagination and critical analysis. … I think such success as I've had in the work I've done in the field of government probably has a good deal to do with that – because they don't catch me up very often in theories of logic in bills, or different parts of bills, that don't hang together.

Moore's third – and final – student at Penn enrolled in the university's graduate program the very year the first two departed. Moore recognized the ability and – more important for his method – the potential of Anna M. Mullikin (1893-1975) at once. She made such vast strides in her first year under his tutelage that she presented a paper on her results at an AMS meeting in October 1919, just one month after beginning her second year of studies at Penn. When asked to recall the Moore Method, the one element that stood out in her mind was R. L. Moore's insistence that students settle questions without assistance of any sort, whether that assistance was supplied by other humans or written sources. She recalled [**22**, pp. 87]:

> He had his work all published … and people would go and look it up in the library and he didn't want them to do that. He wanted them to work it out themselves. And he put them out of class if he found out that they were cheating. (In one class) there were three of us, but he put everybody out but me. One was a Catholic nun and she tried to get help from me and he put her out. He said if she needed help she didn't belong in his class.

Although Mullikin had made substantial progress on her dissertation before Moore moved to Texas at the end of her second year, she opted to accompany him instead of remaining behind, even though J. R. Kline had accepted a position at Penn that became available when Moore vacated the premises. On the one hand Miss Mullikin's choice militates against the absolute independence of students under the Moore system. On the other, she was following the tradition of spending extra time with the master. By the end of the summer of 1921 the dissertation was almost complete, so she returned to Penn to finish it. J. R. Kline arranged a half scholarship to help defray the cost of tuition and then took care of the administrative details necessary for the awarding of her Ph.D. degree in June 1922.

We might mention in passing that J. R. Kline spent the summer of 1921 in Austin with the hopes of continuing his valued collaboration with R. L. Moore. However, Moore was a visiting professor in California during most of that time. Nonetheless the two did manage to get together for a couple weeks. Such collaboration was especially desirable in light of Kline's journeys between Philadelphia and Austin, which included boat excursions between New York and Galveston with train rides on each end between seaport and university.

Anna Mullikin serves as a transitional figure in the Moore Method from its developmental stage to one of complete maturity. But she does not represent the last connection between R. L. Moore and Philadelphia.

Maturity

We have seen that although J. R. Kline shepherded Anna Mullikin's credentials through the graduation process leading up to her Ph.D. in 1922, R. L. Moore was designated as her dissertation supervisor. While she was working under Moore at Texas the preceding year, Moore himself was busy refining his teaching method on a new batch of students in a completely different locale. His first doctoral student, Raymond L. Wilder, earned his Ph.D. in 1923 only one year after Mullikin.

Renke G. Lubben was Moore's next doctoral student, graduating in 1925. Lubben was awarded a National Research Council (NRC) fellowship for postdoctoral study in Europe for the academic year 1926-1927. He chose to work in Göttingen because it housed arguably the most accomplished mathematics faculty in the world. Another factor influencing the choice was that J. R. Kline was already in Göttingen as one of the first contingent of Guggenheim Fellows. In informing Moore of his selection, Kline boasted proudly, "The Guggenheims made 15 appointments … mine was the only appointment in mathematics." [**A3**] Subsequent Kline-to-Moore correspondence from abroad reported on Lubben's activities, thus illustrating the supportive bond that was developing among Moore School participants.

During this time period Moore directed one of his undergraduate students, William L. Ayres, to Penn because he felt that Ayres would benefit more from J. R. Kline. He did, earning his Penn Ph.D. in 1927. When John H. Roberts completed his dissertation under Moore two years later he spent a postdoctoral year working with Kline at Penn before settling at Duke, where he remained for the rest of his career. The Moore-Kline continuum also proceeded in the reverse direction, with Ayres returning to Texas on an NRC fellowship for a year after obtaining his degree. Three other Kline students spent a postdoctoral year with Moore – Harry Gehman (Ph.D. 1925), Norman Rutt (1928), and Leo Zippin (1929). It should be noted, however, that R. L. Moore did not devote much time to postdoctoral students, focusing his energies instead on developing the skills of the talented neophytes whose programs of study he was directing.

One such talent was Gordon T. Whyburn, who became Moore's first Texas-born graduate when he earned his Ph.D. in 1927. Whyburn would later develop an excellent graduate program at the University of Virginia in coordination with his colleague E. J. McShane. What is germane here, however, is the role that Whyburn played in the Moore-Kline continuum. His dissertation made critical use of a result due to Anna Mullikin on four separate occasions. [**24**, pp. 397-400] Following the lead of his mentor, the first American to cite her work in print in 1925 [**15**, p. 421], Whyburn referred to it as "a theorem of Miss Mullikin". (The Polish mathematician Stefan Mazurkiewicz had singled out her work one year before Moore. See [**8**].) The Moore paper of 1925 and the Whyburn paper of 1927, based on his dissertation, helped publicize the result in this country so much that just four years later Moore's initial Texas student R. L. Wilder preceded a reformulation of the theorem with a standard phrase favored by many mathematicians, "it is well known that." [**25**, p. 40]

In the meantime, J. R. Kline's student Leo Zippin was analyzing and extending Miss Mullikin's results as part of his research toward his doctorate. The title of Zippin's 1929 dissertation, "A study of continuous curves and their relation to the Janiszewski-Mullikin theorem", reflects the fact that her major result had been obtained independently by the Polish mathematician Zygmunt Janiszewski. Zippin's dissertation was published in the *Transactions* later that year. [**27**] Another article by Zippin the next year shortened the Janiszewski-Mullikin Theorem to JMT, an abbreviation that suggests yet another level of acceptance. [**28**, p. 339]

Mullikin herself played a decisive role in one other connection between R. L. Moore and J. R. Kline. In spite of the worldwide acclaim her paper received, she remained a teacher at Germantown High School in Philadelphia. When one of her brightest pupils, Mary-Elizabeth Hamstrom, received her diploma in 1944, Mullikin directed her to J. R. Kline at Penn. By the time Hamstrom received her undergraduate degree four years later, she had given little thought to life after the B.A. Therefore when Kline recommended that she pursue graduate studies at Texas she was somewhat flabbergasted, exclaiming it was "a thought that hadn't occurred to me." [**4**, p. 295] Hamstrom did apply to the University of Texas and was admitted at once. Wanting to be fully prepared for the beginning of classes in September 1948 she wrote to R. L. Moore in late April, asking what material she should read over the summer. His response has become the classic document describing the philosophy of the Moore Method, as evidenced by its inclusion among the documents published to celebrate the 100th annual meeting of the American Mathematical Society. [**4**]

One final incident cemented the bond between J. R. Kline and R. L. Moore before Kline's untimely death in 1955. Kline had become chair of the Department of Mathematics at Penn in 1940. Eight years later he hired one of Moore's outstanding graduates, R. D. Anderson. A few years later this hiring turned out to be serendipitous for Lida Barrett, who was working on her doctorate at Penn under Kline in the early 1950s after having studied under R. L. Moore at Texas. Although Kline suggested Barrett's thesis problems, and she essentially solved them under his supervision, he suffered a heart attack that prevented him from seeing the project to completion. So R. D. Anderson took over and helped Barrett polish and refine her dissertation, which she completed in 1954. Barrett later became president of the MAA, a position also held by five other Moore students – R. L. Wilder, G. S. Young, R. H. Bing, E. E. Moise, and R. D. Anderson.

Conclusion

The Anderson-Barrett connection, following on the heels of the Hamstrom-Mullikin trail, brings R. L. Moore's relationship with the University of Pennsylvania full circle. Moore developed his famous teaching method at Penn during the second decade of the 20th century with his first three doctoral students – J. R. Kline, George Hallett, and Anna Mullikin. The exchange of postdoctoral students between Moore and Kline in the 1920s, the close connection of doctoral dissertations written under Moore at Penn and at Texas in the 1920s, and the involvement of both Kline and Moore students in Mullikin's theorem attest to the full maturation of the Moore Method by 1932. As we have seen, the Moore Method was a completed entity by then. (It might be pointed out that Moore directed 41 of his 50 doctoral students after the appearance of his trailblazing book in 1932. [**16**]) The events that took place in the 1950s reflect the extent to which Penn played a central role in the Moore Method even after Moore left Philadelphia.

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