Arthur S. Hathaway and the Theory of Numbers

Although Arthur Stafford Hathaway (1855-1934) studied and researched mathematics at the height of the development of research universities in America, there is very little known about his life (for an overview of the significant events of Hathaway's life mentioned in this paper, see the accompanying timeline.) It is known that he was born in 1855 and died in 1934 [**6**]. In 1879, Hathaway received his B.S. [**8**], though it is unclear where or in what discipline. It is likely that he did his undergraduate research at Johns Hopkins, because Parshall mentions that he was in James Joseph Sylvester's number theory class in 1879 [**3**, p.121]. However, the first clear piece of information regarding his academic studies comes from a list of mathematics fellows at Johns Hopkins University, where Hathaway appears as a fellow from 1881 until 1883 [**3**, p.97]. He in fact spent the years 1880 – 1884 studying at the graduate level at Johns Hopkins University [**7**].

Johns Hopkins University was the first university devoted to the ideal of research and emphasizing graduate education [**3**, p. 53]. It was designed with the idea that all faculty members would teach fewer hours than the traditional load because they would be expected to carry out original research and to train future researchers [**4**, p.2]. Johns Hopkins officially opened in 1876, so Arthur Hathaway was one of the first students to attend classes. Other mathematicians to emerge from Sylvester's group of graduate students at Johns Hopkins were Thomas Craig, Fabian Franklin, and Christine Ladd.

A big influence on the development of Hathaway's research was his teacher, James Joseph Sylvester. Sylvester was an Englishman who had done many years of successful research before taking the position of Professor at Johns Hopkins (he was sixty-one years old in 1876!) [**4**, p.3]. Sylvester taught at Johns Hopkins from its opening until 1883, when he took on a job teaching at Oxford in his native country [**4**, p.8]. Zitarelli comments, "All Hopkins graduate students worked in areas that were of interest to Sylvester … During this period [his students] published papers in … invariant theory, geometry, number theory, matrix theory, and logic" [**4**, p.6].

Arthur Hathaway's *American Journal of Mathematics* article, "Some papers on the theory of numbers," is one example of a paper published during Sylvester's period of influence. Hathaway notes in the beginning of the article that he founded the papers on the principles he learned while attending the classes of Professor J. J. Sylvester on the Theory of Numbers at Johns Hopkins University in 1879 - 1880. In May of 1880, and January, February, and March of 1881, Hathaway presented the papers before the Mathematical Seminary, and then picked them up again for publication in 1884. In the article, he credits Sylvester for his research, saying, "It is due to Professor Sylvester to state that the beginnings of my knowledge in the Theory of Numbers were obtained entirely from short-hand notes of his lectures; and that it was his suggestive presentation of the Theory of Congruences that led to the development of these principles" [**2**, p. 316].

The first of the two papers in Hathaway's "Some papers on the theory of numbers" that appears, and the one on which we shall focus our attention, is titled "On integer classes." Hathaway begins the paper by stating the three fundamental principles of division: (1) if two numbers are separately divisible, so is their sum and difference, (2) if a term is divisible, so is a multiple of it, and (3) if a product is divisible and one factor is relatively prime to the divisor, then the remaining factor is divisible. Hathaway goes on to say that any combination of sums and products of terms retains its remainder with respect to a given divisor, no matter how one

replaces the terms of the combination by other terms that have the same remainders. Thus, if one groups together as a class all terms which have the same remainder (when dividing by a given divisor), the results of any combination of sums and products of classes constitute members of one and the same class. Here, the product of two classes refers to the assemblage of all possible products between any of the terms of the two classes.

Hathaway next asserts that what corresponds to division among remainder classes is the linear modular equation $AX = B$. Here A and B are fixed remainder classes modulo some given number, and the solution X will be the remainder class that makes that linear modular equation true with respect to the given modulus. When restricting A to be a class whose elements are relatively prime to the modulus, Hathaway says that there will only be one solution class X.

He then goes on to discuss the existence of cycles of classes, i.e., those classes which are defined by the property that the members of a cycle are repeated over and over again when multiplied repeatedly by any one member of the cycle. He introduces the concept of a "repetent," which he defines as an element whose powers repeat itself for successive involutions. He notes,

> If the multiplications of each member of a cycle by a given term leave one member unaltered, it will leave all unaltered; for we may first exhibit (by multiplication) the unaltered member as a factor of all, then perform the multiplication. If the multiplier is a member it does not alter itself, and is therefore a repetent. Any member raised to the power represented by the number of members is a repetent; for it leaves the product of all the members (which is a member) unaltered. [**2**, p. 319]

These results are very similar to those that one might study in abstract algebra.

Hathaway continues by generalizing his findings, first to a linear combination of congruences, such as $A_1X_1 + A_1X_2 + \ldots = B$, where A_i are of the form $(k_1k_2\ldots)/k_i$ with $k_1, k_2, \ldots$ being relatively prime factors of the modulus. He then generalizes his results to a binomial congruence $X^n = A$, where n is a fixed number, A is a given remainder class with respect to some modulus, and the solution X will be a remainder class with respect to the modulus. He even goes so far as to mention Galois' Theorem (years before Galois' work became popular) and suggests that there may be imaginary solutions to modular equations.

Hathaway was researching at a high level of number theory when he wrote the paper mentioned above, and was clearly strongly influenced by his professor, J. J. Sylvester. After receiving his Ph.D. from Johns Hopkins University, Hathaway became an instructor at Cornell University. He was an instructor from 1885 until 1890, and an assistant professor 1890-1891 [**8**]. While at Cornell, Hathaway taught courses in number theory, differential equations, and quaternions and vector analysis [**6**]. When one considers the history of Cornell, it is not at all surprising that Hathaway was hired to teach there. When the Board of Trustees of Johns Hopkins' estate was trying to determine how to run the university that was to be built, Andrew D. White (the President of Cornell University) sat on its board of advisers [**3**, p.54]. Thus the President of Cornell had a hand in shaping Johns Hopkins into a research university, and would most likely be interested in hiring its graduates. Also, the Professor of Mathematics at Cornell, Oliver White, sought in the 1880s to "allow his department time for more than just teaching; he wanted them to have the opportunity to do and lead research" [**6**]. For that reason, a recent graduate of the first research university in the country would be an ideal choice for White's faculty. The next of Hathaway's publications occurred while he was teaching at Cornell; his "A memoir in the theory of numbers" appeared in the *American Journal of Mathematics* in 1887.

After leaving Cornell, Hathaway published at least three more times. He wrote *A Primer of Quaternions* in 1896, at which time he is referred to as a Professor of Mathematics at the Rose

Polytechnic Institute in Terre Haute, Indiana. The primer is written to be a textbook for his students. In the Preface to *A Primer of Quaternions*, Hathaway states, "The present work is the outcome of lectures that I have given to my classes for a number of years past as the equivalent of the usual instruction in the analytical geometry of space. The main features of this primer were therefore developed in the laboratory of my classroom…" [**1**]. Hathaway also published "Quaternion space" in the *Transactions of the American Mathematical Society* in 1902, and a book called *Analytical Dynamics being a Synopsis of Leading Topics in the Analytical Theory of Dynamics with Numerous Examples and Selections from Newton's Principia and Other Sources* in 1906, on which he is again listed as Professor of Mathematics at Rose Polytechnic Institute in Terre Haute, Indiana.

Rose Polytechnic Institute, now called Rose-Hulman Institute of Technology, was founded in 1874, under the name Terre Haute School Institute of Technology. Its founder, Chauncey Rose, founded it as an engineering college because he had trouble getting men with technical training to come so far west and remain until the railroad he was constructing was completed [**6**]. Rose "envisioned a school to prepare young men for the mechanical and design demands and opportunities of the Industrial Revolution" [**10**]. One should note the contrast between the engineering college Rose Polytechnic Institute and the research universities of Johns Hopkins and Cornell. It is strange that a researcher such as Hathaway would eventually end up teaching at a technical school instead of continuing with research. However, according to Zitarelli, this was not at all unusual for the students of J. J. Sylvester. Zitarelli asserts "within a short period of time [of Sylvester's departure for England] the fledgling community that had flocked to Sylvester began to disintegrate, with all three of the other professors leaving Johns Hopkins by 1900… So by the turn of the twentieth century, the vital element, the community that had formed around Sylvester, had dissipated" [**4**, p.9].

Written Sources:

1. Hathaway, Arthur S. *A Primer of Quaternions,* 1896. (ftp://ibiblio.org/pub/docs/books/gutenberg/etext06/pqtrn10p.pdf)
2. Hathaway, Arthur S., Some papers on the theory of numbers, *American Journal of Mathematics* **6** (No. 1/4; 1883 – 1884), 316-330.
3. Parshall, Karen Hunger and Rowe, David E. *The Emergence of the American Mathematical Research Community, 1876-1900: J. J. Sylvester, Felix Klein, and E. H. Moore*, Amer. Math. Soc., Providence RI, 1994.

4.) Zitarelli, David "Emergence, 1876-1900" Class notes, Spring 2004.

Online Sources:

5. http://www.hti.umich.edu/cgi/t/text/text-idx?c=umhistmath;idno=ABR5018#resultstoc
6. http://homepages.culver.edu/Faculty/gcochell/CornellArticle.htm
7. http://www.math.cornell.edu/General/History/historyP5.html
8. http://www.math.cornell.edu/~oliver/History/hewett.html
9. http://www.rose-hulman.edu/echos/125th/chaunceyrose.htm
10. http://www.rose-hulman.edu/HR/New%20Staff%20Handbook/New%20Staff%20Handbook.htm

Emily C. Warner
May 6, 2004

Timeline of Known events of Arthur Stafford Hathaway's Life