Erratum to: “A Proof of Tsygan’s Formality Conjecture for an Arbitrary Smooth Manifold”

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Abstract
Boris Shoikhet noticed that the proof of lemma 1 in section 2.3 of [1] contains an error. In this note I give a correct proof of this lemma which was kindly suggested to me by Dmitry Tamarkin. The correction does not change the results of [1].

1 Introduction

In this note I give a correct proof of lemma 1 from section 2.3 in [1]. This proof was kindly suggested to me by Dmitry Tamarkin and it is based on the interpretation of $L_\infty$-morphisms as Maurer-Cartan elements of an auxiliary $L_\infty$-algebra.

The notion of partial homotopy proposed in section 2.3 in [1] is poorly defined and this note should be used as a replacement of section 2.3 in [1]. The main result of this section (lemma 1) is used in section 5.2 of [1] in the proof of theorem 6. Since the statement of the lemma still holds so does the statement of theorem 6 as well as all other results of [1].

In section 2 of this note I recall the notion of an $L_\infty$-algebra and the notion of a Maurer-Cartan element. In section 3, I give the interpretation of $L_\infty$-morphisms as Maurer-Cartan elements of an auxiliary $L_\infty$-algebra and use it to define homotopies between $L_\infty$-morphisms. Finally, in section 4 I formulate and prove lemma 1 from section 2.3 of [1].

Notation. I use the notation from [1]. The underlying symmetric monoidal is the category of cochain complexes. For this reason I sometimes omit the combination “DG” (differential graded) talking about (co)operads and their algebras. For an operad $\mathcal{O}$ I denote by $F_{\mathcal{O}}$ the corresponding Schur functor. $s\ K$ denotes the suspension of the complex $K$. In other words,

$$s\ K = s \otimes K,$$

where $s$ is the one-dimensional vector space placed in degree $+1$. Similarly,

$$s^{-1}\ K = s^{-1} \otimes K,$$

where $s^{-1}$ is the one-dimensional vector space placed in degree $-1$. $\text{cocomm}$ is the cooperad of cocommutative coalgebras.

By “suspension” of a (co)operad $\mathcal{O}$ I mean the (co)operad $\Lambda(\mathcal{O})$ whose $m$-th space is

$$\Lambda(\mathcal{O})(m) = \sum_{1}^{m} \mathcal{O}(m) \otimes \text{sgn}_m,$$

where $\text{sgn}_m$ is the sign representation of the symmetric group $S_m$.

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2 \( L_\infty \)-algebras and Maurer-Cartan elements

Let me recall from [4] that an \( L_\infty \)-algebra structure on a graded vector space \( \mathcal{L} \) is a degree 1 codifferential \( Q \) on the colagebra \( \mathbb{F}_{\text{Acocomm}}(\mathcal{L}) \) cogenerated by \( \mathcal{L} \). Following [1] I denote the DG coalgebra \((\mathbb{F}_{\text{Acocomm}}(\mathcal{L}), Q)\) by \( C(\mathcal{L}) \):

\[
C(\mathcal{L}) = (\mathbb{F}_{\text{Acocomm}}(\mathcal{L}), Q). \tag{2.1}
\]

A morphism \( F \) from an \( L_\infty \)-algebra \((\mathcal{L}, Q)\) to an \( L_\infty \)-algebra \((\mathcal{L}^\circ, Q^\circ)\) is by definition a morphism of (DG) coalgebras

\[
F : C(\mathcal{L}) \to C(\mathcal{L}^\circ). \tag{2.2}
\]

Since

\[
\mathbb{F}_{\text{Acocomm}}(\mathcal{L}) = s \mathbb{F}_{\text{cocomm}}(s^{-1} \mathcal{L})
\]

the vector space of \( C(\mathcal{L}) \) can be identified with the exterior algebra \( \wedge^* \mathcal{L} \) and for a graded vector space \( V \) a map

\[
f : \mathbb{F}_{\text{Acocomm}}(\mathcal{L}) \to V
\]

of degree \(|f|\) can be identified with the infinite collection of maps

\[
f_n : \mathcal{L}^\otimes n \to V, \quad n \geq 1,
\]

where each map \( f_n \) has degree \(|f| + 1 - n\) and

\[
f_n(\ldots, \gamma, \gamma', \ldots) = (-1)^{|\gamma||\gamma'|} f_n(\ldots, \gamma', \gamma, \ldots)
\]

for every pair of elements \( \gamma, \gamma' \in \mathcal{L} \).

Due to proposition 2.14 in [3] every coderivation of \( \mathbb{F}_{\text{Acocomm}}(\mathcal{L}) \) is uniquely determined by its composition with the projection

\[
\text{pr}_\mathcal{L} : \mathbb{F}_{\text{Acocomm}}(\mathcal{L}) \to \mathcal{L} \tag{2.3}
\]

from \( \mathbb{F}_{\text{Acocomm}}(\mathcal{L}) \) onto cogenerators.

In particular, the codifferential \( Q \) of the coalgebra \( C(\mathcal{L}) \) is uniquely determined by the infinite collection of maps

\[
Q_n = \text{pr}_\mathcal{L} \circ Q \big|_{\wedge^n \mathcal{L}} : \wedge^n \mathcal{L} \to \mathcal{L}, \tag{2.4}
\]

such that \( Q_n \) has degree \( 2 - n \). In [1] \( Q_n \) are called structure maps of the \( L_\infty \)-algebra \( \mathcal{L} \).

The equation \( Q^2 = 0 \) is equivalent to an infinite collection of quadratic equations on the maps (2.4). The precise form of these equations can be found in definition 4.1 in [2].

One of the obvious equations implies that the structure map of the first level \( Q_1 \) is a degree 1 differential of \( \mathcal{L} \). Thus an \( L_\infty \)-algebra can be thought of as an algebra over an operad in the category of cochain complexes.

If \( \mathcal{L} \) is a pronilpotent \( L_\infty \)-algebra then it makes sense to speak about its Maurer-Cartan elements:

**Definition 1 (Definition 4.3 in [2])** A Maurer-Cartan \( \pi \) of a pronilpotent \( L_\infty \)-algebra \((\mathcal{L}, Q)\) is a degree 1 element of \( \mathcal{L} \) satisfying the equation

\[
\sum_{n=1}^{\infty} \frac{1}{n!} Q_n(\pi, \pi, \ldots, \pi) = 0. \tag{2.5}
\]
Let me remark that the infinite sum in (2.5) is well defined since \( L \) is pronilpotent.

Every Maurer-Cartan element \( \pi \) of \( L \) can be used to modify the \( L_\infty \)-algebra structure on \( L \). This modified structure is called the \( L_\infty \)-structure twisted by the Maurer-Cartan \( \pi \) and its structure maps are given by

\[
Q^n_\pi(\gamma_1, \ldots, \gamma_n) = \sum_{m=1}^{\infty} \frac{1}{m!} Q^n_{m+n}(\pi, \ldots, \pi, \gamma_1, \ldots, \gamma_n), \quad \gamma_i \in L.
\] (2.6)

It is equation (2.5) which implies that the maps (2.6) define an \( L_\infty \)-algebra structure on \( L \).

Two Maurer-Cartan elements \( \pi_0 \) and \( \pi_1 \) are called equivalent if there is an element \( \xi \in L^0 \) such that the solution of the equation

\[
\frac{d}{dt}\pi_t = Q^\pi_1(\xi)
\] (2.7)

connects \( \pi_0 \) and \( \pi_1 \):

\[
\pi_t \bigg|_{t=0} = \pi_0, \quad \pi_t \bigg|_{t=1} = \pi_1.
\]

3 \( L_\infty \)-morphisms and their homotopies

I will need the following auxiliary statement:

**Proposition 1** Let \( \mathcal{O} \) be an operad and \( A \) be an algebra over \( \mathcal{O} \). If \( B \) is a (DG) cocommutative coalgebra then the cochain complex

\[
\mathcal{H}_{B,A} = \text{Hom}(B, A)
\] (3.1)

of all linear maps from \( B \) to \( A \) has a natural structure of an algebra over \( \mathcal{O} \).

**Proof.** The \( \mathcal{O} \)-algebra structure on \( A \) is by definition the map (of complexes)

\[
\mu_A : \mathcal{F}_\mathcal{O}(A) \to A
\] (3.2)

making the following diagrams commutative:

\[
\begin{array}{ccc}
\mathcal{F}_\mathcal{O}(\mathcal{F}_\mathcal{O}(A)) & \xrightarrow{\mathcal{F}_\mathcal{O}(\mu_A)} & \mathcal{F}_\mathcal{O}(A) \\
\downarrow \mu_\mathcal{O}(A) & & \downarrow \mu_A \\
\mathcal{F}_\mathcal{O}(A) & \xrightarrow{\mu_A} & A,
\end{array}
\] (3.3)

\[
\begin{array}{ccc}
A & \xrightarrow{u_\mathcal{O}(A)} & \mathcal{F}_\mathcal{O}(A) \\
\downarrow \text{id} & & \downarrow \mu_A \\
A & &
\end{array}
\] (3.4)

where \( \mu_\mathcal{O} \) and \( u_\mathcal{O} \) are the transformation of functors

\[
\mu_\mathcal{O} : \mathcal{F}_\mathcal{O} \circ \mathcal{F}_\mathcal{O} \to \mathcal{F}_\mathcal{O},
\]

\[
u_\mathcal{O} : \text{Id} \to \mathcal{F}_\mathcal{O}
\]
defined by the operad structure on \( \mathcal{O} \). The map \( \mu_A \) is called the multiplication.
For every $n > 1$ the comultiplication $\Delta$ in $B$ provides me with the following map

$$\Delta^{(n)} : B \to B^\otimes n$$

$$\Delta^{(n)} X = (\Delta \otimes 1^\otimes (n-2)) \cdots (\Delta \otimes 1 \otimes 1)(\Delta \otimes 1)\Delta X$$  \hspace{1cm} (3.5)

Using this map and the $O$-algebra structure on $A$ I define the $O$-algebra structure on $\mathcal{H}_{B,A}$ (3.1) by

$$\mu(v, \gamma_1, \ldots, \gamma_n; X) = \mu_A(v)[\gamma_1 \otimes \cdots \otimes \gamma_n (\Delta^{(n)} X)] ,$$  \hspace{1cm} (3.6)

where $v \in O(n) , \gamma_i \in \text{Hom}(B, A)$ , and $X \in B$ .

The equivariance with respect to the action of the symmetric group follows from the cocommutativity of the comultiplication on $B$ .

The commutativity of the diagram

$$\begin{array}{ccc}
\mathcal{F}_O(\mathcal{F}_O(\mathcal{H}_{B,A})) & \xrightarrow{\mathcal{F}_O(\mu)} & \mathcal{F}_O(\mathcal{H}_{B,A}) \\
\downarrow \mu_O(\mathcal{H}_{B,A}) & & \downarrow \mu \\
\mathcal{F}_O(\mathcal{H}_{B,A}) & \xrightarrow{\mu} & \mathcal{H}_{B,A}
\end{array} \hspace{1cm} (3.7)$$

follows from the commutativity of (3.3) and the associativity of the comultiplication in $B$ .

The commutativity of the diagram

$$\begin{array}{ccc}
\mathcal{H}_{B,A} & \xrightarrow{u_O(\mathcal{H}_{B,A})} & \mathcal{F}_O(\mathcal{H}_{B,A}) \\
\downarrow \text{id} & & \downarrow \mu \\
\mathcal{H}_{B,A} & \xrightarrow{\mu} & \mathcal{H}_{B,A}
\end{array} \hspace{1cm} (3.8)$$

and the compatibility of $\mu$ (3.6) with the differential are obvious. \hspace{1cm} \square

Since

$$\mathcal{F}_{\text{cocomm}}(L) = s \mathcal{F}_{\text{cocomm}}(s^{-1}L)$$

for every $L_\infty$-algebra $L^\circ$ proposition 1 gives me a $L_\infty$-structure on the cochain complex

$$\mathcal{U} = s \text{Hom}(C(L), L^\circ) .$$  \hspace{1cm} (3.9)

This algebra $\mathcal{U}$ can be equipped with the following decreasing filtration:

$$\mathcal{U} = \mathcal{F}^1\mathcal{U} \supset \mathcal{F}^2\mathcal{U} \supset \cdots \supset \mathcal{F}^k\mathcal{U} \supset \cdots$$

$$\mathcal{F}^k\mathcal{U} = \{ f \in \text{Hom}(\bigwedge^k L, L^\circ) \mid f\big|_{\bigwedge^{<k} L} = 0 \} .$$  \hspace{1cm} (3.10)

It is not hard to see that this filtration is compatible with the $L_\infty$-algebra structure on $\mathcal{U}$ . Furthermore, since $\mathcal{U} = \mathcal{F}^1\mathcal{U}$ , for every $k$ the $L_\infty$-algebra $\mathcal{U}/\mathcal{F}^k\mathcal{U}$ is nilpotent. On the other hand,

$$\mathcal{U} = \lim_k \mathcal{U}/\mathcal{F}^k\mathcal{U} ,$$  \hspace{1cm} (3.11)

and hence, the $L_\infty$-algebra $\mathcal{U}$ is pronilpotent and the notion of a Maurer-Cartan element of $\mathcal{U}$ makes sense.

My next purpose is identify the Maurer-Cartan elements of the $L_\infty$-algebra $\mathcal{U}$ (3.9) with $L_\infty$-morphisms from $L$ to $L^\circ$ .
Proposition 2 \(L_{\infty}\)-morphisms from \(L\) to \(L^\circ\) are identified with Maurer-Cartan elements of the \(L_{\infty}\)-algebra \(U\) (3.9)

Proof. Since \(C(L^\circ)\) is a cofree coalgebra, the map \(F\) (2.2) is uniquely determined by its composition \(pr_{L^\circ} \circ F\) with the projection \(pr_{L^\circ}\) (2.3). This composition is a degree zero element of \(\text{Hom}(C(L), L^\circ)\). Thus, since \(U\) (3.9) is obtained from \(\text{Hom}(C(L), L^\circ)\) by the suspension, every morphism \(F\) (2.2) is identified with a degree 1 element of \(U\).

It remains to prove that the compatibility condition

\[Q^\circ F = FQ\] (3.12)

of \(F\) with the codifferentials \(Q\) and \(Q^\circ\) on \(C(L)\) and \(C(L^\circ)\), respectively, is equivalent to the Maurer-Cartan equation (2.5) on \(pr_{L^\circ} \circ F\) viewed as an element of \(U\).

It is not hard to see that

\[pr_{L^\circ} \circ (Q^\circ F - FQ) = 0.\] (3.13)

is equivalent to the Maurer-Cartan equation on the composition \(pr_{L^\circ} \circ F\) viewed as an element of \(U\) (3.9).

Thus, I have to show that equation (3.13) is equivalent to the compatibility condition (3.12).

For this, I denote by \(\Psi\) the difference:

\[\Psi = Q^\circ F - FQ\]

and remark that

\[\Delta \Psi = - (\Psi \otimes F + F \otimes \Psi) \Delta,\] (3.14)

where \(\Delta\) denotes the coproduct both in \(C(L)\) and \(C(L^\circ)\).

The latter follows from the fact that \(Q\) and \(Q^\circ\) are coderivations and \(F\) is a morphism of cocommutative coalgebras.

Given a cooperad \(C\), a pair of cochain complexes \(V, W\), a degree zero map

\[f : V \to W\]

and an arbitrary map

\[b : V \to W\]

I denote by \(\partial(b, f)\) the following map\(^1\)

\[\partial(b, f) : F_C(V) \to F_C(W)\]

\[\partial(b, f)(\gamma, v_1, v_2, \ldots, v_n) = \]

\[(-1)^{|b|(|\gamma| + |v_1| + \cdots + |v_{i-1}|)} \sum_{i=1}^{n} (\gamma, f(v_1), \ldots, f(v_{i-1}), b(v_i), f(v_{i+1}), \ldots, f(v_n)),\] (3.15)

\[\gamma \in C(n), \quad v_i \in V,\]

where \(|\gamma|, |b|, |v_j|\) are, respectively, degrees of \(\gamma\), \(b\), and \(v_j\). The equivariance of (3.15) with respect to permutations is obvious.

\(^1\)A similar construction was introduced at the beginning of section 2.2 in [3].
It is not hard to see that condition (3.14) is equivalent to commutativity of the following diagram

\[
\begin{array}{ccc}
F_{\Lambda}^{\text{cocomm}}(L) & \xrightarrow{\Psi} & F_{\Lambda}^{\text{cocomm}}(L^\circ) \\
\downarrow \nu & & \downarrow \nu \\
F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L)) & \xrightarrow{\partial(\Psi,F)} & F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L^\circ)),
\end{array}
\]

(3.16)

where \(\nu\) is the coproduct of the cotriple \(F_{\Lambda}^{\text{cocomm}}\).

Since the functor \(F_{\Lambda}^{\text{cocomm}}\) with the transformations \(\nu : F_{\Lambda}^{\text{cocomm}} \to F_{\Lambda}^{\text{cocomm}} \circ F_{\Lambda}^{\text{cocomm}}\) and \(\text{pr} : F_{\Lambda}^{\text{cocomm}} \to \text{Id}\) form a cotriple\(^2\), the following diagram

\[
\begin{array}{ccc}
F_{\Lambda}^{\text{cocomm}}(L^\circ) & \xrightarrow{\partial(\Psi,F)} & F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L^\circ)) \\
\downarrow \nu & \searrow \text{id} & \\
F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L^\circ)) & \xrightarrow{p} & F_{\Lambda}^{\text{cocomm}}(L^\circ),
\end{array}
\]

(3.17)

with \(p\) being \(F_{\Lambda}^{\text{cocomm}}(\text{pr}_{L^\circ})\), commutes.

Attaching this diagram to (3.16) I get the commutative diagram

\[
\begin{array}{ccc}
F_{\Lambda}^{\text{cocomm}}(L) & \xrightarrow{\Psi} & F_{\Lambda}^{\text{cocomm}}(L^\circ) \\
\downarrow \nu & & \downarrow \nu \\
F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L^\circ)) & \xrightarrow{\partial(\Psi,F)} & F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L^\circ)) \\
\downarrow \nu & \searrow \text{id} & \\
F_{\Lambda}^{\text{cocomm}}(F_{\Lambda}^{\text{cocomm}}(L^\circ)) & \xrightarrow{p} & F_{\Lambda}^{\text{cocomm}}(L^\circ),
\end{array}
\]

(3.18)

where, as above, \(p = F_{\Lambda}^{\text{cocomm}}(\text{pr}_{L^\circ})\).

Hence,

\[\Psi = F_{\Lambda}^{\text{cocomm}}(\text{pr}_{L^\circ}) \circ \partial(\Psi,F) \circ \nu.\]

On the other hand

\[F_{\Lambda}^{\text{cocomm}}(\text{pr}_{L^\circ}) \circ \partial(\Psi,F) = \partial(\text{pr}_{L^\circ} \circ \Psi, \text{pr}_{L^\circ} \circ F).\]

Therefore,

\[\Psi = \partial(\text{pr}_{L^\circ} \circ \Psi, \text{pr}_{L^\circ} \circ F) \circ \nu\]

and \(\Psi\) vanishes if and only if so does the composition \(\text{pr}_{L^\circ} \circ \Psi\).

This concludes the proof of the proposition. \(\square\)

The identification proposed in the above proposition allows me to introduce a notion of homotopy between two \(L_\infty\)-morphisms. Namely,

**Definition 2** \(L_\infty\)-morphisms \(F\) and \(\tilde{F}\) from \(L\) to \(L^\circ\) are called homotopic if the corresponding Maurer-Cartan elements of the \(L_\infty\)-algebra \(U\) (3.9) are equivalent.

### 4 Lemma 1 from [1]

Let me denote by \(F_n\) the components

\[F_n : \wedge^n L \to L^\circ\]

\(^2\text{See, for example, section 1.7 in [3].}\)
\[
F_n = \text{pr}_{\mathcal{L}^\circ} \circ F \bigg|_{\wedge^n \mathcal{L}} \tag{4.1}
\]
of the composition \(\text{pr}_{\mathcal{L}^\circ} \circ F\), where \(\text{pr}_{\mathcal{L}^\circ}\) is the projection from \(\mathcal{F}_{\text{Acocomm}}(\mathcal{L}^\circ)\) onto cogenerators. In [1] the maps (4.1) are called structure maps of the \(L_\infty\)-morphism (2.2).

The compatibility condition (3.12) implies that the structure map \(F_1\) of the first level is morphism of complexes:

\[
F_1 : \mathcal{L} \to \mathcal{L}^\circ, \quad Q_1^\circ F_1 = F_1 Q_1.
\]

By definition, an \(L_\infty\)-morphism \(F\) is a \(L_\infty\)-quasi-isomorphism if the map \(F_1\) is a quasi-isomorphism of the corresponding complexes.

I can now prove the following lemma:

**Lemma 1** Let

\[
F : C(\mathcal{L}) \mapsto C(\mathcal{L}^\circ)
\]
be a quasi-isomorphism from an \(L_\infty\)-algebra \((\mathcal{L}, Q)\) to an \(L_\infty\)-algebra \((\mathcal{L}^\circ, Q^\circ)\). For \(n \geq 1\) and any map

\[
\tilde{H} : \wedge^n \mathcal{L} \mapsto \mathcal{L}^\circ \tag{4.2}
\]
of degree \(-n\) one can construct a quasi-isomorphism

\[
\tilde{F} : C(\mathcal{L}) \mapsto C(\mathcal{L}^\circ)
\]
such that for any \(m < n\)

\[
\tilde{F}_m = F_m : \wedge^m \mathcal{L} \mapsto \mathcal{L}^\circ \tag{4.3}
\]
and

\[
\tilde{F}_n(\gamma_1, \ldots, \gamma_n) = F_n(\gamma_1, \ldots, \gamma_n) + Q_1^\circ \tilde{H}(\gamma_1, \ldots, \gamma_n) - (-)^n \frac{\partial}{\partial \tau} \tilde{H}(\gamma_1, \ldots, Q_1^\circ(\gamma_n)), \tag{4.4}
\]
where \(\gamma_i \in \mathcal{L}^{k_i}\).

**Proof.** Let \(Q^\mathcal{L}\) denote the \(L_\infty\)-algebra structure on \(\mathcal{U}\) (3.9). Let \(\alpha\) be the Maurer-Cartan elements of \(\mathcal{U}\) corresponding to the \(L_\infty\)-morphism \(F\).

By setting

\[
\xi_{\wedge^m \mathcal{L}} = \begin{cases} \tilde{H}, & \text{if } m = n, \\ 0, & \text{otherwise} \end{cases} \tag{4.5}
\]
I define an element \(\xi \in \mathcal{U}\) of degree \(-1\). By definition of the filtration (3.10) the element \(\xi\) belongs to \(\mathcal{F}^n \mathcal{U}\).

Let \(\alpha_t\) be the unique path of Maurer-Cartan elements defined by

\[
\frac{d}{dt} \alpha_t = (Q^\mathcal{L})^\alpha_t(\xi), \quad \alpha_t \bigg|_{t=0} = \alpha. \tag{4.6}
\]

The unique solution \(\alpha_t\) of (4.6) can be found by iterating the following equation in degrees in \(t\)

\[
\alpha_t = \alpha + \int_0^t (Q^\mathcal{L})^\alpha_t(\xi) d\tau. \tag{4.7}
\]
Since the \(L_\infty\)-algebra \(\mathcal{U}\) is pronilpotent the recurrent procedure (4.7) converges.
It is not hard to see that, since $\xi \in \mathcal{F}^n \mathcal{U}$,
\begin{equation}
\alpha_t - \alpha \in \mathcal{F}^n \mathcal{U}
\end{equation}
and
\begin{equation}
\alpha_t - (\alpha + tQ^U_1(\xi)) \in \mathcal{F}^{n+1} \mathcal{U}.
\end{equation}

Let $\tilde{F}$ be the $L_\infty$-morphism from $\mathcal{L}$ to $\mathcal{L}^\circ$ corresponding to the Maurer-Cartan element
\[ \tilde{\alpha} = \alpha_t \bigg|_{t=1} . \]

Equation (4.8) implies (4.3) and equation (4.9) implies (4.4). It is obvious that, since $F$ is a quasi-isomorphism, so is $\tilde{F}$.

The lemma is proved. □

References


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