Since we have a sequence of quasi-isomorphisms of DGAs connecting polyvectors $P$ with cochains $C^{*+1}(A)$ we have a correspondence between equivalence classes of formal Poisson structures
\[ \overline{\Pi} = \partial \Pi_1 + \hbar^2 \Pi_2 + \cdots \] 
and the star products $\star$ on $K[[x_1, \ldots, x_n]]$. 

Lecture 16
Construction of the star-product.
It turns out that using \( N_k \)'s we can get an explicit formula for the \( \pi \).

Exercise 9.2 Let \((\xi, \eta, [,])\) be a DGLA. Show that if \( \omega \) is a MC element of \( \xi \) then

\[
\beta = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} \underbrace{\omega \omega \ldots \omega}_{k \text{ times}}
\]

is a MC element of \( \text{Lie}(sC(s^2E)) \).

Solution with a gap: The equation \( \frac{d}{dt} + \frac{1}{2} [\,]_E = 0 \) implies that \( Q \beta = 0 \) (This is the gap). Therefore

\( Q \beta = (-1)^{\frac{k}{2}} [\, , ] \Delta \beta = \ldots \)
\[ = \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} \sum_{\begin{subarray}{c} p+q=k \\ p,q \geq 1 \end{subarray}} \left[ \alpha^p, \alpha^q \right] \cdot \frac{k!}{p!q!} = \]

\[ = \frac{1}{2} \sum_{p,q \geq 1} (-1)^{p+q+1} \left[ \alpha^p, \alpha^q \right] = -\frac{1}{2} \left[ \beta, \beta \right]. \]

Thus, \( \partial \beta + \frac{1}{2} \left[ \beta, \beta \right] = 0. \)

It is also clear that \( \partial \alpha(\beta) = 0. \)

Thus a formal Poisson structure lifts to a MC element of \( \text{Lie} (\mathfrak{g}(s^{-1} \mathbb{P}^n))[[t]] \)

\[ \beta_{\Pi} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} \Pi \text{ times} \]

\[ = 3 \]
Applying $U: \text{Lie}(\mathfrak{s}(\mathfrak{s}^* P))) \to C^{-\infty}(\mathfrak{a})$ we get the star-product:

$$\alpha \ast \beta = \alpha \cdot \beta + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} U_k(T_1 \ldots T_k)(\alpha, \beta).$$