Assignment 8

Exercise 1. (4 pts.) Let $B$ be a (coassociative) coalgebra and $Q_1, Q_2$ be coderivations of $B$ with degrees $q_1$ and $q_2$, respectively. Prove that the commutator

$$[Q_1, Q_2] = Q_1 \circ Q_2 - (-1)^{q_1 q_2} Q_2 \circ Q_1$$

is a coderivation of $B$.

Exercise 2. (3 pts.) Let $\text{Vect}$ be the category of $K$-vector spaces. Consider the monad $(T, \mu, \varepsilon)$ corresponding to the functor $T$ which sends a vector space $V$ to its tensor algebra

$$T(V) = K \oplus V \oplus V^{\otimes 2} \oplus V^{\otimes 3} \oplus \ldots.$$ 

Prove that algebras over the monad $(T, \mu, \varepsilon)$ are unital associative algebras.

Exercise 3. (3 pts.) Let $\text{Vect}$ be the category of $K$-vector spaces. Consider the comonad $(C, \Delta, p)$ corresponding to the functor $C$ which sends a vector space $V$ to the truncated symmetric algebra

$$C(V) = V \oplus S^2(V) \oplus S^3(V) \oplus \ldots.$$ 

Here $\Delta$ is the “comultiplication”:

$$\Delta_V : C(V) \to CC(V)$$

and the “counit” $p$ is the projection

$$p_V : C(V) \to V.$$ 

Prove that coalgebras over the comonad $(C, \Delta, p)$ are cocommutative (coassociative) algebras (without counit).