Assignment 13

Exercise 1. (5 pts.) Let $M$ be a smooth real manifold and $v, w$ be polyvector fields on $M$. For a coordinate neighborhood $U \subset M$ we set
\[
[v, w]_S |_U := [v|_U, w|_U]_S
\]
where the bracket $[\cdot, \cdot]_S$ in the right hand side is the Schouten bracket we used for $\mathbb{R}^d$. Prove that the right hand side of (1) transforms correctly under a change of local coordinates. Thus, equation (1) defines a Lie bracket on $V^*_{M^{\bullet+1}}$.

Hint: You may start with the case when $v$ and $w$ are vector fields. In this case the Schouten bracket coincides with the usual Lie bracket of vector fields.

Exercise 2. (5 pts.) Let $\omega$ be a non-degenerate 2-form on $M$ and $\pi$ be a bivector field defined by the formula
\[
\sum_j \pi^{ij}(x)\omega_{jk}(x) = \delta^i_k.
\]
Prove that $\pi$ is a Poisson bivector field $\Leftrightarrow \omega$ is closed.

Hint: A 2-form $\omega$ is closed $\Leftrightarrow$
\[
\partial_x^i \omega_{jk}(x) + \partial_{x^j} \omega_{ki}(x) + \partial_{x^k} \omega_{ij}(x) = 0
\]
for all triples of indices $i, j, k$. 