Assignment 11

The goal of this assignment is to prove partially the following statement.

**Theorem 1** If \( n \geq 1 \), then the assignment

\[
(p_1, \ldots, p_n, q_1, \ldots, q_k) \mapsto \left( \text{Arg}(p_i - p_j), \text{Arg}(\bar{p}_i - p_j), \text{Arg}(p_i - q_l), \frac{q_l - q_{l1}}{q_l - q_{l2}}, \frac{|p_i - p_{j1}|}{|p_i - p_{j2}|}, \frac{\text{Im}(p_i)}{\text{Im}(p_j)} \right)
\]  

\[ i \neq j; i \neq j_1, i \neq j_2, j_1 \neq j_2; l \neq l_1, l \neq l_2, l_1 \neq l_2 \]

defines a smooth embedding of \( C_{n,k} \) into the space

\[
(S^1)^{2n(n-1)} \times ([0, -\pi])^nk \times \mathbb{R}^{k(k-1)(k-2)} \times ([0, \infty])^{n(n-1)(n-2)} \times ([0, \infty])^{n(n-1)}.
\]

**Exercise 1.** (6 pts.) Prove that the (smooth) functions

\[
\left\{ \text{Arg}(p_i - p_j), \text{Arg}(\bar{p}_i - p_j), \text{Arg}(p_i - q_l), \frac{q_l - q_{l1}}{q_l - q_{l2}}, \frac{|p_i - p_{j1}|}{|p_i - p_{j2}|}, \frac{\text{Im}(p_i)}{\text{Im}(p_j)} \right\}
\]

separate points on \( C_{n,k} \). In other words, if the above functions take the same values on two points \( T_1, T_2 \in C_{n,k} \) then \( T_1 = T_2 \). *Thus you prove that map (1) is one-to-one.*

**Exercise 2.** (4 pts.) Consider map (1) for \( C_{2,0} \). Prove that the differential of map (1) has rank = 2 at every point of \( C_{2,0} \).