Generalized Bochner Formulas and Ricci Lower Bounds for sub-Riemannian Manifolds of Rank Two

Abstract: This talk describes recent joint work with Fabrice Baudoin. We introduce a new class of sub-Riemannian manifolds of rank two which encompasses Riemannian manifolds, CR manifolds with vanishing Webster-Tanaka torsion, orthonormal bundles over Riemannian manifolds, and graded nilpotent Lie groups of step two. These manifolds admit a canonical horizontal connection and a canonical sub-Laplacian. We construct on these manifolds an analogue of the Riemannian Ricci tensor and prove Bochner type formulas for the sub-Laplacian. As a consequence, it is possible to formulate on these spaces a sub-Riemannian analogue of the so-called curvature dimension inequality. Sub-Riemannian manifolds for which this inequality is satisfied are shown to share many properties in common with Riemannian manifolds whose Ricci curvature is bounded from below.