\(\Omega' = \{ \omega : \exists n_1, \ldots, n_n \to \infty \, \omega \in A_n(\omega) \} \)

\( p(D) = 1 \)

\( \Omega' \) has property

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**Occupation Times**

**Def** Let \( D = B(0, r) = \{ y : |y| < r \} \) then the occupation time for \( B_t \) is

\[ \int_0^\infty 1_D(B_t) \, dt \]

i.e. the total length of the time in \( D \), possibly \( \infty \)

\( \Omega_m \) let \( D = B(0, r) \) then

2.1) \( P_x \left( \int_0^\infty 1_D(B_t) \, dt = \infty \right) = 1 \) in \( d \leq 2 \)

2.2) \( E_{D_k} \left( \int_0^\infty 1_D(B_t) \, dt = \infty \right) < \infty \) in \( d \geq 3 \)

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\[ T_0 := 0 \]
\[ S_k := \inf \{ t > T_{k-1} : B_t \in D \} \]
\[ T_k := \inf \{ t > S_k : B_t \notin G \} \]

*Strong Markov property remark.*

By strong Markov property for \( k \geq 1 \), let \( T = T_1 \).

\[ P_x \left( \int_{S_k}^{T_1} 1_D(B_t) \, dt \geq \gamma \mid \mathcal{F}_{S_k} \right) = P_x \left( \int_0^{T_1} 1_D(B_t) \, dt \geq \gamma \right) = H(\gamma) \]

*Recall.*

If \( Z_t \) adapted to \( G_t \) and \( f \) bounded \& measurable,

\[ E\left[ f(Z_t) \mid G_0 \right] = E\left[ f(Z_{t-1}) \right] \]

Then \( Z_t \) has stationary independent increments.

Do Thm 2.8 + proof + summary up to §3.3 today.