A continuous random variable $X$ has its probability density function $f(x)$ and cumulative distribution function $F(x)$.

For discrete distributions:

- $P(X=0) = \frac{1}{4}$
- $P(X=1) = \frac{1}{2}$
- $P(X=2) = \frac{1}{4}$

The cumulative distribution function $F(x)$ is given by:

$$F(x) = P(X \leq x) = \begin{cases} 
0 & x < 0 \\
\frac{1}{4} & 0 \leq x < 1 \\
\frac{3}{4} & 1 \leq x < 2 \\
1 & x \geq 2 
\end{cases}$$

The random variable $U_1$ is used to select values $X_1 < X_2 < X_3 < \ldots$ from a sample space according to the cumulative distribution function $F(x)$.

- $F(X_{j-1}) \leq U_1 < F(X_j)$

With random variables $X_1 < X_2 < X_3 < \ldots$, $X = X_j$ is the smallest value of $X_1, \ldots, X_n$ in the sample space.

- $F(0) = \frac{1}{4} \leq U_1 < F(1) = \frac{3}{4}$

So $1$ is selected.

- $F(0) \leq U_1 = \frac{1}{2} < F(1)$

Whereas $F(1) \leq U_1 = \frac{3}{4} < F(2)$

$2$ selected.

Values of random variable $X_1 < X_2 < X_3 < \ldots$ are selected such that $F(X_{j-1}) \leq U_1 < F(X_j)$. 

- $F(X_{j-1}) \leq U_1 < F(X_j)$
\( P(X = x) = \frac{1}{5} \quad x = 1, 2, 3, 4, 5 \) Uniform discrete variable

\[
F(x) = \begin{cases} 
0 & x < 1 \\
\frac{1}{5} & 1 \leq x < 2 \\
\frac{2}{5} & 2 \leq x < 3 \\
\frac{3}{5} & 3 \leq x < 4 \\
\frac{4}{5} & 4 \leq x < 5 \\
1 & 5 \leq x 
\end{cases}
\]

Remember \( P_r(X = x) = F(x) - \lim_{x \to x^-} F(x) \)

Eq: \( P_r(x = 2.5) = F(2.5) - F(2.5^-) = 0 \)

Exercise

\( X \sim \text{B}(n, p) \)

0 \leq U < 0.2 \quad \text{select} \quad x = 0

0.2 \leq U < 0.4 \quad \text{select} \quad x = 1

0.4 \leq U < 0.6 \quad \text{select} \quad x = 2

0.6 \leq U < 0.8 \quad \text{select} \quad x = 3

0.8 \leq U \quad \text{select} \quad x = 4

\[
\binom{n}{k} p^k (1-p)^{n-k}
\]

\[
\binom{n}{k} (1-p)^{n-k} = 0.027
\]

\[
\binom{n}{k} p^k = 1 - 0.657 = 0.343
\]

\[
\log p = \log 0.343
\]

\[
n \log (1-p) = 0.027
\]
\[ F(x) = \begin{cases} 
0.5x & , 0 \leq x < 1 \\
0.5 + 0.25x & , 1 \leq x \leq 2 
\end{cases} \]

\[ P(X = 1) = F(1) - F(1^-) = 0.75 - 0.5 = 0.25 \]

Read Ex 17.3