\[ N(+) \sim \mathcal{P}(2^+) \rightarrow \text{Type I: } P \rightarrow N_1(+) \sim \mathcal{P}(\lambda_1) \]
\[ \lambda_1 = 2 \cdot P \]
\[ \text{Type II: } 1 - P \rightarrow N_2(+) \sim \mathcal{P}(\lambda_2) \]
\[ \lambda_2 = 2(1 - P) \]

\[ S_{n_1}^I \quad (n_1 \text{ events of Type I}) \]
\[ S_{n_2}^II \quad (n_2 \text{ events of Type II}) \]

\[ \text{Want to find: } P(S_{n_1}^I < S_{n_2}^II) \]
\[ P(\text{Type I happens before Type II}) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \]
\[ P(\text{Type II happens before Type I}) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \]

If we observe \( n_1 + n_2 - 1 \) events and at most \((n_2 - 1)\) events of type II happen, then

\[ X \sim B(n_1 + n_2 - 1, P) \]
\[ \lambda_2 = \frac{\lambda_2}{\lambda_1 + \lambda_2} \text{ by above} \]

Thus,

\[ P(S_{n_1}^I < S_{n_2}^II) = \sum_{k=0}^{n_2-1} \frac{(n_1 + n_2 - 1)!}{k!(n_1 + n_2 - 1 - k)!} \left( \frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^k \left( 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{n_1 + n_2 - 1 - k} \]

A key result.
\[ X = \text{Loss} = \nu(0, 1000) \]

\[ \# \text{ of losses by time } t, \ N(t) \sim \mathcal{P}(\lambda t), \ \lambda = 1/10 \text{/year}. \]

Claim: If loss exceeds the deductible at $400, then no claim.

(a) How many claims expected in 2 years?

Type I: \[ P[\text{Loss} > 400] = \frac{400}{1000} = 0.4 \Rightarrow \text{Claim Made.} \]

Type II: \[ P[\text{Loss} < 400] = 0.6 \Rightarrow \text{No Claim.} \]

\[ N_1(t) : \# \text{ of claims per time } t \sim \mathcal{P} (\lambda t), \ \lambda = 2 \cdot 0.6 = 10 \cdot 0.6 \]

(We don't care about \( N_2(t) \) here)

\[ \Rightarrow \sim \mathcal{P}(6t) \]

\[ \therefore N_1(2) \sim \mathcal{P}(6 \cdot 2) = \mathcal{P}(12) \]

\[ \Rightarrow E[N_1(2)] = 12 \]

(b) What is the mean time between claims, call it \( T_1 \)

\[ E[T_1] ? = \frac{1}{\lambda} = \frac{1}{6} \quad \text{from (a)} \]
c) Find out prob. that 5 partially covered losses occur before 4 uncovered losses.

\[ P \left[ S_5^I < S_4^U \right] \]

This is asking in 5+4-1=8 events, the prob of at most 3 events of Type II occur.

\[ P \left[ S_5^I < S_4^U \right] = \sum_{k=0}^{3} \frac{8!}{k! \left( 8-k \right)!} \left( \frac{4}{10} \right)^k \left( \frac{6}{10} \right)^{8-k} \]

Ex: Verify Markov formula, \( P(S_3^I < S_4^U) \) is the same (equiv) to one in class

Ex: Problems 19, 21, 22, 23

For Grads:

\[ P \left[ S_3 > + \right] = P \left[ N(+) \leq 2 \right] \]

up to point \((0, +)\), the 3rd event has not occured.

\[ E \left[ S_3 \right] = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \quad \text{with} \quad S_3 = T_1 + T_2 + T_3 \]

- \( E \left[ S_3 | N(1) = m \right] = 2 \) \( \Rightarrow \) with \( m \), \( N(1) = m \)

i) \( P \left[ S_3 > + | N(1) = 2 \right] = P \left[ N(+) \leq 2 \right] | N(1) = 2 \)
\[ P \left[ N(t) - N(1) \leq 2 | N(1) = 2 \right] \]

# of poisson events between time 1 and t
also note: they are independent by ind. inc. property.

\[ P \left[ N(t) - N(1) = 0 \right] = P \left[ N(t-1) = 0 \right] \]

then we can say \( N(t-1) \sim P(z(t-1)) \)

thus \( P[N(t-1) = 0] = e^{-2(t-1)} \)

ii) \( P(S_3 < t | N(1) = 2) = 1 - e^{-2(t-1)} \) for \( t > 1 \)

iii) \( E[S_3 | N(1) = 2] = 2 e^{2(t-1)} \) for \( t > 1 \) (its p.d.f.)

iv) \( E[S_3 | N(1) = 2] = \sum_{t=1}^{\infty} \frac{\mathbb{E}[S_3]}{t!} (t | N(1) = 2) \)

Let \( t-1 = w \)

\[ \sum_{w=0}^{\infty} (1+w) \cdot 2e^{-2w} dw \]

\[ = \sum_{w=0}^{\infty} 2e^{-2w} dw + \sum_{w=0}^{\infty} w2e^{-2w} dw \]

\[ = \left[ 1 + \frac{1}{2} \right] \]

Could have gone right to here by memory less property.

we know 1 unit time elapsed + \( E[S_3 | N(1) = 2] \)

\[ = 1 + \frac{1}{2} \]
For Grades: Prove by method on previous page (Integral)

\[ E[S_n \mid N(t) = m] , n > m \]

Show \( s \rightarrow = t_o + \frac{n-m}{2} \)

Intro to Non-Homogeneous Poisson Process.

\( N(t) \)

\( \lambda = \text{Constant rate of Poisson Events} \)

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 + \\
\hline
\underline{1} & \underline{1} & \underline{1} & \underline{1} \\
\end{array}
\]

Consider \( \lambda(t) \), a function of \( t \). Then we know \( \lambda(t) \) is constant. We call \( \lambda(t) \) the "intensity function".

Under this assumption below,

\[ P[N(t+h) - N(t) = 1] = h \lambda(t) + o(h) \]

\[ P[N(t+s) - N(s) = n] = e^{-[m(t+s)-m(s)]} \cdot \frac{[m(t+s)-m(s)]^n}{n!} \]

with \( m(t) = \int_{0}^{t} \lambda(y) \, dy = \text{Mean Value Function} \).