Problem #11876. Proposed by A. Cibulis, Latvia. Let \( a, b \) be the roots of \( x^2 + x + \frac{1}{2} = 0 \). Find
\[
\sum_{n=1}^{\infty} \frac{(-1)^n(a^n + b^n)}{n + 2}.
\]

Solution by Tewodros Amdeberhan, Tulane University, LA, USA. Notice that \( ab = \frac{1}{2} \) and \( a + b = -1 \). In fact, let \( a = \frac{-1+i}{2} \) and \( b = \frac{-1-i}{2} \) where \( i = \sqrt{-1} \). Since \( |a| = |b| = \frac{1}{\sqrt{2}} \). The series expansions \( \sum_{n \geq 0} a^n z^n = \frac{1}{1-az} \) and \( \sum_{n \geq 0} b^n z^n = \frac{1}{1-bz} \) are valid for \( |z| < \sqrt{2} \). In this domain, we may add and safely integrate (using path-independence of analytic functions)
\[
\sum_{n \geq 0} \frac{(a^n + b^n)(-1)^{n+2}}{n + 2} = \int_0^{-1} \sum_{n \geq 0} (a^n + b^n)z^{n+1} \, dz = \int_0^{-1} \left( \frac{z}{1-az} + \frac{z}{1-bz} \right) \, dz
\]
\[
= \int_0^{-1} \left( 2z - \frac{(a+b)z^2}{abz^2 - (a+b)z + 1} \right) \, dz = \int_0^{-1} \left( \frac{2z + z^2}{\sqrt{2}z^2 + z + 1} \right) \, dz
\]
\[
= [2z - 4 \arctan(z + 1)]_{-1}^{-1} = -2 + \pi.
\]

That means, the required sum is \( \sum_{n \geq 1} \frac{(-1)^n(a^n + b^n)}{n + 2} = -1 + \sum_{n \geq 0} \frac{(-1)^n(a^n + b^n)}{n + 2} = \pi - 3. \) \( \square \)