MORE ON PASCAL-TYPE TRIANGLES ...

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Abstract.

K. Dilcher and K.B. Stolarsky [DS] considered Pascal-type triangles to characterize twin primes. They also ask [DS, page 680] "whether there are triangles ... that characterize prime pairs with fixed differences larger than two." They added "... we were unable to find an analogue ... with the required properties."

Our first theorem extends D’Angelo’s [D] single prime identification, while the second theorem provides a family generalizing the triangle constructions in [DS] and gives an answer to the above question. In the sequel, $t$ remains an even positive integer.

**Theorem 1**: The integer $tk + 1$ is prime iff $tk + 1$ divides

$$\left( \binom{k + (t - 1)s}{ts} \frac{tk + 1}{ts + 1} \right)$$

for all $0 \leq s \leq k$, except exactly one entry (or, remove $tk + 1$ then all are integers but one).

**Theorem 2**: (prime gap $t$) The integer pair $(tk(k-1) + 3, tk(k-2) + 3)$ is $t$-gap prime pair iff each of the numbers $tk(k-1) + 3, tk(k-2) + 3$ divides the $k$th row of the triangle

$$\left( \binom{k + (t - 1)s}{ts + 1} \frac{(tk(k-1) + 3)(tk(k-2) + 3)}{ts + 3} \right)$$

for all $0 \leq s \leq k - 1$, except exactly one entry (i.e. remove $tk(k-1) + 3$ (resp. $tk(k-2) + 3$) then all are integers but one).

References
