1 April 2010

Homework 8

Theoretical Linear Algebra 3051 **Daniel B. Szyld**

Due Tuesday 6 April 2010.

1. Give an example of a (non–trivial) 3×3 matrix, and two different Schur decompositions for it.

2. Give an example of a space V with an inner product, and an invertible map $T \in \mathcal{L}(V)$ such that $T^{-1} \neq T^*$.

3. Give an example of an operator $T \in \mathcal{L}(V)$ such that $\|T\| = 1$, and another operator $A \in \mathcal{L}(V)$, $A \neq 0$ with $\|A\| < 1$.

4. *

   Let A be self–adjoint and $v \neq 0$. Consider a Krylov subspace of dimension m, i.e., $\mathcal{K}_m(A, v) = span\{v, Av, A^2v, \ldots, A^{m-1}v\}$ in $\mathbb{R}^n$. Show how to construct an orthonormal basis $\{v_1, v_2, \ldots, v_m\}$ of $\mathcal{K}_m(A, v)$ using a three-term recurrence. Prove that indeed the way you suggest gives an orthonormal basis.