PROBLEM SET 6

Theoretical Linear Algebra 3051 **Daniel B. Szyld**

Due Tuesday March 23

1. Consider the subspaces U and W of $V = \mathcal{P}_3(\mathbb{R})$, defined by
$U = \{a(t+1)^2 + b \mid a, b \in \mathbb{R}\}$ and $W = \{a + bt + (a+b)t^2 + (a-b)t^3 \mid a, b \in \mathbb{R}\}$.
   (i) Show that $V = U \oplus W$.
   (ii) Find a basis for $U^\perp$, for some appropriate inner product.

2. On $V = \mathbb{R}^3$ we consider the inner product $\langle , \rangle$ defined by

$$\langle (x_1, y_1, z_1), (x_2, y_2, z_2) \rangle = x_1x_2 + 2y_1y_2 + 3z_1z_2,$$

for all (x_1, y_1, z_1) and (x_2, y_2, z_2) in V. Let $u = (1, 2, -1)$ and $v = (3, -2, 0)$.
   (i) Compute $\langle u, v \rangle$, $\|u\|$, $\|v\|$ and the "cosine" of the "angle" between u and v.
   (ii) Find a basis for $u^\perp$.

3. Using the Schmidt orthonormalization process, change the $\mathbb{R}^3$-basis

$$\mathcal{B} = \{v_1 = (1, 0, 1), v_2 = (0, 1, 1), v_3 = (1, 1, 0)\}$$

into an orthonormal one $\mathcal{C} = \{w_1, w_2, w_3\}$. Write the vector $u = (1, -2, 7)$ as a linear combination of elements in $\mathcal{C}$.

In addition, do exercises 21 and 22 from Axlers' book (p. 124).