1. Consider the subspaces $U$ and $W$ of $V = \mathcal{P}_3(\mathbb{R})$, defined by

\[ U = \{ a(t+1)^2 + b \mid a, b \in \mathbb{R} \} \quad \text{and} \quad W = \{ a + bt + (a+b)t^2 + (a-b)t^3 \mid a, b \in \mathbb{R} \}. \]

(i) Show that $V = U \oplus W$.

(ii) Find a basis for $U^\perp$, for some appropriate inner product.

2. On $V = \mathbb{R}^3$ we consider the inner product $\langle \cdot, \cdot \rangle$ defined by

\[ \langle (x_1, y_1, z_1), (x_2, y_2, z_2) \rangle = x_1x_2 + 2y_1y_2 + 3z_1z_2, \]

for all $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ in $V$. Let $u = (1, 2, -1)$ and $v = (3, -2, 0)$.

(i) Compute $\langle u, v \rangle$, $\|u\|$, $\|v\|$ and the “cosine” of the “angle” between $u$ and $v$.

(ii) Find a basis for $u^\perp$.

3. Using the Schmidt orthonormalization process, change the $\mathbb{R}^3$-basis

\[ B = \{ v_1 = (1, 0, 1), v_2 = (0, 1, 1), v_3 = (1, 1, 0) \} \]

into an orthonormal one $\mathcal{C} = \{ w_1, w_2, w_3 \}$. Write the vector $u = (1, -2, 7)$ as a linear combination of elements in $\mathcal{C}$.

In addition, do exercises 21 and 22 from Axlers' book (p. 124).