Problem Set 5

(Out Tue 11/01/2022, Due Thu 11/10/2022)

Problem 6

Write a staggered grid finite difference method for the Stokes problem

$$\begin{cases} -\nabla^2 \vec{u} + \nabla p = \vec{f} & \text{in } \Omega \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega \\ \vec{u} = \vec{g} & \text{on } \partial\Omega \end{cases}$$

and apply the code to the channel flow problem on $\Omega =]0,1[^2$ with forcing $\vec{f} = 0$, and the following b.c. at the inflow: $\vec{g}(0,y) = (5y^2(1-y)^2, 0)$, and the outflow: $\vec{g}(1,y) = (y(1-y), 0)$, and no-slip wall conditions: $\vec{g}(x,0) = (0,0) = g(x,1)$.

Then modify your code to solve the mixed Dirichlet/Neumann problem instead, where the b.c. $\vec{u} = \vec{g}$ on $\partial\Omega$ is replaced by specifying the inflow conditions $\vec{g}(0,y) = (5y^2(1-y)^2, 0)$, and wall conditions $\vec{g}(x,0) = (0,0) = \vec{g}(x,1)$, but now imposing homogenous Neumann b.c. at the outflow: $\frac{\partial \vec{g}}{\partial x}(1,y) = 0$.

For both cases, plot the resulting velocity and pressure fields, and moreover visualize the difference between the results of the two problems.

You are encouraged to use the code `mit18086_navierstokes.m` as well as previous problem set solutions as a starting point.

Problem P2

The task of this practice problem is for the assigned team of students to get a finite element package running, and to prepare several representative and interesting examples of incompressible fluid flow problems, including the Stokes problem and the time-dependent Navier-Stokes equations. Suggested options are deal.II (`https://www.dealii.org`) or FEniCS (`https://fenicsproject.org/`). The results should be demonstrated in a short presentation (15 minutes) to the rest of the class.