Temple Math 5044 **Introduction to Numerical Analysis II** **Spring 2019**

Problem Set 6

(Out Wed 03/13/2019, Due Mon 03/25/2019)

Problem 6

Consider the r-step BDF method of the form

$$\sum_{j=0}^{r} \alpha_j U^{n+j} = k f(U^{n+r})\ . \tag{1}$$

(a) Write a short Matlab program that for each r automatically computes the vector of coefficients $\vec{\alpha} = (\alpha_0, \dots, \alpha_r)$, such that (1) is globally r-th order accurate.

(b) Plot the regions of absolute stability for the BDF methods $r \in \{1, \dots, 8\}$.

(c) What can you say about zero-stability?

(d) Do the regions of absolute stability grow or shrink with increasing order?

(e) Apply all 8 schemes (BDF1 to BDF8, using exact starting values) to the test problem

$$\begin{cases} \vec{u}'(t) = A \cdot \vec{u}(t) \\ \vec{u}(0) = \mathring{\vec{u}} \end{cases}$$

where

$$A = \begin{pmatrix} 161 & 581 & -999 & -580 \\ -181 & -401 & 999 & 780 \\ 19 & -181 & -1 & -200 \\ -19 & 181 & -999 & -800 \end{pmatrix} \quad \text{and} \quad \mathring{\vec{u}} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\ .$$

on $t \in [0, 1]$, and plot the error convergence (error vs. step size in log–log) for 200 k-values within $10^{-3} \leq k \leq 10^{-1}$. Explain your observations, in particular the humps in BDF4, BDF5, and BDF6.

(f) Can you find an 8-step method (i.e., $r = 8$) of the form (1) that is 7th order accurate and zero-stable? If yes, plot its region of absolute stability.

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where `1` stands for the problem number and `a` for the sub-problem letter.