Temple 9200 **Computational Methods for Flow Problems** **Spring 2015**

Problem Set 6

(Out Wed 03/25/2015, Due Wed 04/08/2015)

Problem 7

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a) Download the two Matlab files `mit18086_fd_transport_limiter.m` and `temple8024_weno_claw.m` from the course website `http://math.temple.edu/~seibold/teaching/2015_9200/` . Explain what these two numerical schemes do, their similarities, and their differences.

b) Adapt the file `temple8024_weno_claw.m` to solve the traffic flow problem from Problem 7 (previous problem set). Email your code under the name `yourfamilyname_problem7b.m` . For a resolution of $\Delta x = 0.1$, which code yields a more accurate result? The WENO code, or the second order *Clawpack* code?

c) Based on these two files, write your own code that implements a second-order MUSCL scheme, using a piecewise linear reconstruction in space, and a semi-discrete time-stepping (using Heun's method). Apply your code to the linear advection equation $\rho_t + \rho_x = 0$ on the domain $x \in [0, 1]$ with periodic b.c., initial conditions $\rho(x, 0) = \sin(2\pi x) + \chi_{[\frac{1}{4},\frac{3}{4}]}(x)$, and final time $t = 2$. Implement a choice of the following limiter functions: (a) none (upwind); (b) none (Lax-Wendroff), (c) superbee, (d) van-Leer, (e) minmod. Email your code under the name `yourfamilyname_problem7c.m` . Using 100 grid cells, which choice of limiter yields the most accurate result?

Problem 8

Consider the linear advection equation $\rho_t + \rho_x = 0$ on the domain $x \in [0, 1]$ with periodic b.c., and $t \in [0, 1]$. Use your code from Problem 7c with the following schemes: upwind, Lax-Wendroff, and minmod limiter. Determine numerically the L^1 convergence rates (with $\Delta t = 0.9\Delta x$) of the three methods for the following sets of initial conditions:

a) $\rho(x, 0) = \sin(2\pi x)$

b) $\rho(x, 0) = \begin{cases} x & x \in [0, \frac{1}{2}) \\ 1 - x & x \in [\frac{1}{2}, 1) \end{cases}$

c) $\rho(x, 0) = \begin{cases} 1 & x \in [0, \frac{1}{2}) \\ 0 & x \in [\frac{1}{2}, 1) \end{cases}$

Explain your observations.