Temple 9200 **Computational Methods for Flow Problems** **Spring 2015**

Problem Set 1

(Out Fri 01/16/2015, Due Wed 01/28/2015)

Problem 1

Download the Matlab file `temple9200_velocity_field.m` from the course website
`http://math.temple.edu/~seibold/teaching/2015_9200/`
and run it.

(a) Explain what you see, i.e., what does the code do?

(b) Remove the moving particles from the code and create a different program (using the same time-dependent velocity field) that plots the velocity field's

- streamlines (using a reasonable choice of how many and which ones to plot);
- streaklines (draw two of them, starting at two different points chosen so that interesting streaklines arise);
- pathlines (again, draw two of them, starting at two points that yield interesting curves);

In order to understand what these lines are, read the `wikipedia` article on "Streamlines, streaklines, and pathlines", linked on the course website. Submit your code in a new Matlab file named `yourfamilyname_problem1b.m` by email to the course instructor. In addition, include a plot of the output of your code at the final time $t = 5$ in your paper submission.

(c) Create (and submit by email) a new Matlab file `yourfamilyname_problem1c.m` based on the original example file. Now we focus on the Lagrangian particles. Adapt the existing code so that your file produces a plot of the solution to the advection equation

$$\rho_t + \vec{v} \cdot \nabla \rho = 0 \ ,$$

where $\vec{v}(x, y, t)$ is the velocity field given in the code, and the initial density is

$$\rho(x, y, 0) = \begin{cases} 1 & (x, y) \in [0.1, 0.5] \times [0.4, 0.8] \\ 0 & \text{otherwise} \end{cases}$$

At any given time t, plot the density $\rho(x, y, t)$ as a function of x and y, either as a 3d plot (using `mesh` or `surf`), or as a color plot (using `imagesc`). Submit a printout of the solution at $t = 5$, with a fine enough resolution (i.e., enough particles and grid resolution) so that one can see the small features of the solution.

(d) Now create (and submit by email) another Matlab file `yourfamilyname_problem1d.m` that conducts everything as in part (c), however now solving the conservative transport equation

$$\rho_t + \nabla \cdot (\vec{v}\rho) = 0 \ .$$

Hint: The task of recovering a density from particles is called "kernel density estimation" (`wikipedia` is your friend). In Matlab, the function `ksdensity` may be helpful.