Temple 3044 Numerical Analysis II Spring 2012

Problem Set 3

(Out Thu 02/02/2012, Due Thu 02/09/2012)

Instructions

- Problems marked with **(T)** are theory problems. Their solutions are to be submitted on paper.
- Problems marked with **(P)** are practical problems, and require the use of the computer. Their solutions are to be submitted on paper, and usually require two parts: (a) a description of the underlying theory; and (b) code segments, printouts of program outputs, plots, and whatever it required to convince the grader that you have understood the theory and addressed all practical challenges appropriately.

Generally, naked numbers are not acceptable. Solutions must include a short write-up describing the problem, your solution technique, and procedural details. To include a computer printout use the cut and paste method for placement of materials in your work. All things must be clearly labeled.

Note: This is only a half problem set, i.e. parts (a)–(c) give only half the usual credit. (You can do (d), (e) for bonus credit.) Either way, you are expected to use the remaining time to work on your project.

Problem C

(T)&(P) Consider the Lotka-Volterra predator-prey model from Problem (B):

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t)) \;, \tag{1}$$

where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\vec{f}(\vec{x}) = \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix} = \begin{pmatrix} x - 4xy \\ -y + 5xy \end{pmatrix}$.

(a) Calculate the Jacobian matrix $D\vec{f}(\vec{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x}(x,y) & \frac{\partial f_1}{\partial y}(x,y) \\ \frac{\partial f_2}{\partial x}(x,y) & \frac{\partial f_2}{\partial y}(x,y) \end{pmatrix}$, and the product $D\vec{f}(\vec{x}) \cdot \vec{f}(\vec{x})$.

(b) Use the expressions from (a) to formulate a second order Taylor method for (1).

(c) Redo the figure from Problem (B) part (d), and add the numerical approximation obtained with the second order Taylor method to the approximation obtained by Euler's method, using the same parameters (please use different colors or labels for the two numerical solutions). Explain your observations.

(d) [Bonus Challenge 1] Use the second order Taylor method developed above to derive the time T it takes for a trajectory $\vec{x}(t)$ to return (for the first time) to its initial value, i.e. $\vec{x}(T) = \vec{x}(0)$. Do this for at least 50 initial values $\vec{x}(0) = \begin{pmatrix} a \\ 0.25 \end{pmatrix}$ with $0 < a < 0.2$, and thus produce a plot of the function $T(a)$.

(e) [Bonus Challenge 2] For $a \approx 0.25$, it takes fairly close to $T = 2\pi$ for the solution to "go around" once. Explain this by linearizing the ODE (1) around the point $\vec{z} = \begin{pmatrix} 0.2 \\ 0.25 \end{pmatrix}$.