Problem 9

The fire control of New South Wales would like to test a new approach to impede the propagation of bush fires: In a checkerboard pattern, regular squares of 1 km × 1 km are sprayed so that the propagation speed of the fire front is slowed down.

1. Write a program that simulates the advance of a fire front that starts in the center of an untreated square and moves outward in its normal direction, with a velocity that is 1 km/h in the untreated squares, and \( \varepsilon \) km/h in the sprayed squares.

2. Create a function \( d(\varepsilon) \), where \( d \) denotes the largest distance of the fire from the origin at the final time \( T_{\text{final}} = 24 \) h. Do so by running your simulation for a whole range of values \( \varepsilon \in [\frac{1}{100}, 1] \). Also plot the shape of the burning region for \( \varepsilon \in \{\frac{1}{100}, \frac{1}{3}, \frac{4}{5}, 1\} \).

3. Explain your results: Are there any critical values of \( \varepsilon \) at which a transition in the fire shape occurs? Is the idea of a checkerboard spraying a good one?

Problem 10

Consider the Korteweg–de Vries (KdV) equation

\[ u_t + 6uu_x + u_{xxx} = 0 \quad \text{on} \quad x \in [-1, 1] \]  

with periodic boundary conditions.

Write a highly accurate spectral code for this equation, and demonstrate its efficiency by running the initial conditions \( u_0(x) = f_{400}(x + 0.7) + f_{200}(x) \) up to time \( t = 0.1 \) (or even further).

Compare this new code with the code that you developed in Problem 8. How much faster is it to achieve the same accuracy?