Temple 8023 **Numerical Differential Equations I** **Spring 2010**

Problem Set 6

(Out Tue 04/13/2010, Due Tue 04/27/2010)

Problem 12

Develop your own numerical scheme for the linear advection equation

$$u_t + u_x = 0 \ .$$

Your scheme must be stable and at least third order accurate in space and in time.

(1) Prove that your scheme satisfies the accuracy requirement.

(2) Prove that your scheme is stable.

(3) Implement your scheme and test it on the example shown in the LeVeque book in Section 10.8.

(4) How does your scheme compare with upwind and Lax-Wendroff?

Problem 13

Consider the advection-reaction equation

$$u_t + u_x = r(u) + s(x, u)$$

on $x \in [0, 2\pi[$ with periodic boundary conditions, and zero initial conditions $u(x, 0) = 0$. The solution $u(x, t)$ represents a chemical concentration ($0 \leq u \leq 1$), which is advected with constant velocity, and modified by a bistable reaction term $r(u) = u(1-u)(u-\frac{1}{2})$ and a localized source term $s(x, u) = a \exp\left(-10(x-\pi)^2\right)(1-u)$, where $a > 0$ is a parameter.

(1) Write a program that approximates the true solution with sufficient accuracy, and run the simulation on the two cases $a = 0.5$ and $a = 1$. Plot both solutions at times $t \in \{2, 8, 40\}$. Explain your observations.

(2) There is a critical threshold value a_c, such that for $a < a_c$, the solution behaves like the case $a = 0.5$, and for $a > a_c$, the solution behaves like the case $a = 1$. Find a_c numerically, up to at least 0.1% accuracy. Remember that your scheme's global truncation error must be sufficiently small.