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The de Haas–van Alphen effect at a quantum critical point

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Abstract

Many heavy-fermion systems are close to a quantum critical point in that, as pressure is varied, the paramagnetic heavy-fermion phase becomes unstable to a magnetically ordered phase at zero temperature. The superconducting phases all occur within a narrow range of pressures centered around the quantum critical point. In materials such as UGe_2, the paramagnetic phase is unstable to a ferromagnetically ordered phase. In this case, the heavy-fermion superconducting phase is contained within the ferromagnetic phase. We examine the effect of quantum critical fluctuations on the heavy quasi-particles, and how these effects are manifested in the de Haas–van Alphen oscillations. It is found that the non-linear field-dependence of the susceptibility becomes vitally important as the quantum critical point is approached, and can give rise to a logarithmic dependence of the quasi-particle mass on the applied field.

1. Introduction

Recently, it has become apparent that many heavy-fermion systems [1,2] are close to a quantum critical point [3]. That is, as the pressure is varied, the paramagnetic heavy-fermion phase becomes unstable to a magnetically ordered phase, at $T \rightarrow 0$. Furthermore, with the exception of UBe_{13}, it has been found that the heavy-fermion superconducting phases all occur within a narrow range of pressures centered around the quantum critical point.

The importance of collective modes associated with the quantum critical point raises questions about the applicability of the single-ion Kondo model as a paradigm for heavy-fermion systems. Previously, it had been often assumed that the large specific heats found in heavy-fermion systems had their origins in large-amplitude local-moment fluctuations similar to those described by the single-impurity Anderson model [4,5]. However, measurements of the de Haas–van Alphen oscillations conclusively show that the mass enhancements are those of a liquid of

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itinerant heavy quasi-particles, which form dispersive bands. Furthermore, the quasi-particles have the same Fermi-surface as expected from electronic structure calculations. In analogy with the suggestion made for the high T_c superconducting cuprates, the heavy Fermi-liquid phase might not survive right up to the quantum critical point but may be replaced by a marginal Fermi-liquid. The question as to whether the collective magnetic fluctuations that occur in the vicinity of the quantum critical point are responsible for the large quasi-particle mass enhancements of heavy-fermion systems is, as yet, unanswered.

The paramagnetic heavy-fermion phase is usually unstable to an anti-ferromagnetic phase, however, in systems such as UGe_2 the system is unstable to a ferromagnetic state [6]. The coexistence of ferromagnetism and superconductivity is surprising, since it is well known that uniform magnetic fields suppress superconductivity. More precisely, if superconductivity and magnetism are to coexist, the superconducting coherence length must be far greater than the length scale over which the internal-magnetic field varies. In the case of UGe_2, the heavy-fermion superconducting phase resides completely within the magnetically ordered phase and the ferromagnetism undergoes a second-order transition at the precise pressure where the superconductivity vanishes. Therefore, it is possible that the low-energy ferromagnetic fluctuations play an important role in mediating the superconducting pairing, and also that the spin-fluctuation pairing mechanism in UGe_2 produce spin-triplet Cooper pairs in analogy to the pairing mechanism in superfluid ^{3}He. Therefore, the study of the effects that ferromagnetic spin-fluctuations have on the quasi-particles at a quantum critical point are of special interest.

In this note, we shall examine the effect that ferromagnetic fluctuations have on the quasi-particle spectrum close to a quantum critical point. In particular, we shall examine the effect of the spin-fluctuations on the quasi-particles of the Fermi-liquid, and how they are manifested in measurements of the de Haas–van Alphen effect.

2. The model

The electronic system at the quantum critical point will be described by the Hubbard model. The Hamiltonian is given by the sum of two terms

$$\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{I}} \tag{1}$$

where $\hat{H}_0$ represents the Hamiltonian for non-interacting electrons in a Bloch band, and $\hat{H}_{\mathrm{I}}$ is a short-ranged Coulomb interaction between pairs of electrons.

The Hamiltonian $\hat{H}_0$ is written as

$$\hat{H}_0 = \sum_{k,\sigma} \varepsilon_k \hat{d}^{\dagger}_{k,\sigma} \hat{d}_{k,\sigma} \tag{2}$$

in which $\hat{d}^{\dagger}_{k,\sigma}$ and $\hat{d}_{k,\sigma}$, respectively, create and destroy an electron of spin σ in a Bloch state labeled by the Bloch wave vector k. The energy of the Bloch state is denoted by ε_k. The screened Coulomb interaction $\hat{H}_{\mathrm{I}}$ is written as:

$$\hat{H}_{\mathrm{I}} = \frac{U}{2}\sum_{k,q,\sigma}\hat{d}^{\dagger}_{k+q,\sigma}\hat{d}^{\dagger}_{k-q,-\sigma}\hat{d}^{\dagger}_{k,-\sigma}\hat{d}^{\dagger}_{k,\sigma} \tag{3}$$

where U is the strength of the screened Coulomb interaction. When treated within the mean-field approximation [7], the paramagnetic phase of this model undergoes an instability to a magnetically ordered phase for a value of U greater than a critical value U_{c}. The nature of the magnetically ordered state and the value of the critical interaction U_{c} can be obtained from the generalized Stoner criterion [8]. This criterion is obtained by examining the response to a static staggered magnetic field. The system becomes unstable to an infinitesimal applied static staggered field of wave vector Q, for a critical value of U_{c} when

$$\chi(Q;0) \to \infty \tag{4}$$

The frequency-dependent susceptibility is to be calculated within the Random Phase Approximation (RPA) [9,10], in which the magnetic fluctuations are treated as Gaussian fluctuations.

In the RPA, the transverse susceptibilities are expressed in terms of multiple-scattering processes involving an up-spin electron with a down-spin hole shown in Fig. 1 [9,10]. This yields the approximation

$$\chi^{+-}(q;\nu) \;=\; \frac{\mu_B^2\chi_0(q;\nu)}{1 - U\chi_0(q;\nu)} \tag{5}$$

where $\chi_0(Q;0)$ is the reduced non-interacting susceptibility first evaluated by Lindhard. $\chi_0(Q;0)$ is given by

$$\chi_0(q;\nu) = \frac{1}{N}\sum_k\left[\frac{f(\varepsilon(k)) - f(\varepsilon(k+q))}{\hbar\nu - \varepsilon(k) + \varepsilon(k+q) + \mathrm{i}\delta}\right] \tag{6}$$

In the RPA, a magnetic instability of the paramagnetic state to an ordered state with ordering wave vector Q is obtained when the static susceptibility $\chi(Q;0)$ diverges, which occurs due to the vanishing of a denominator. This first happen at $T \to 0$ when the generalized Stoner

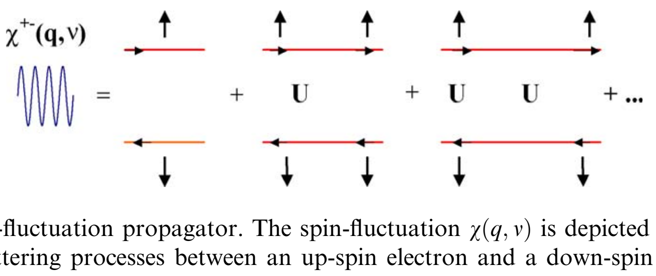

Fig. 1. The transverse spin-fluctuation propagator. The spin-fluctuation $\chi(q,\nu)$ is depicted by the wavy line. It corresponds to the multiple-scattering processes between an up-spin electron and a down-spin hole. The electron propagators are depicted by the directed solid lines. The spin state σ is denoted by an up or down arrow. The Coulomb interaction U is denoted by a dashed vertical line.

criterion is fulfilled

$$1 = U\chi_0(Q;0) \tag{7}$$

The paramagnetic state is unstable for values of U greater than a critical value U_c where the equality of (7) first holds, at any value of Q. The type of instability, determined by Q and the critical value of U at which it occurs, is governed by both the quasi-particle band structure and the state of occupation of the bands. If $\chi_0(Q;0)$ is largest at $Q=0$, the system is expected to become unstable to a ferromagnetic state at a critical value of U determined by the usual Stoner criterion for ferromagnetism, $1 = U_c\rho(\mu)$. For perfect nesting tight-binding bands, such that at half-filling

$$\varepsilon(k+Q) = -\varepsilon(k) \tag{8}$$

one finds that $\chi_0(Q;0)$ diverges for $Q=\pi(1,1,1)$. In this case, the non-interacting susceptibility can be written in terms of an integral over the density of states for the non-interacting Bloch electrons, $\rho(\varepsilon)$,

$$\chi_0(q;\omega) = \int \mathrm{d}\varepsilon\, \rho(\varepsilon)\left[\frac{2f(\varepsilon)-1}{\hbar\nu - 2\varepsilon + \mathrm{i}\delta}\right] \tag{9}$$

The integral diverges logarithmically at the Fermi-energy. This leads to an instability to an anti-ferromagnetically ordered state for U greater than the critical value of $U_c = 0$ [11]. The general phase diagram for the three-dimensional Hubbard model with tight-binding bands has a paramagnetic phase for small values of U, except for close to half-filled bands. At larger values of U and with low electron concentrations, the paramagnetic phase may become unstable to a ferromagnetic phase. As the number of electrons is increased, the ferromagnetic phase gives way to incommensurate spin-density wave phases which subsequently continuously evolve into an anti-ferromagnetic phase at half-filling.

In general, for U values close to the critical value U_c, the static susceptibility evaluated at the relevant Q value is enhanced and the imaginary part of the susceptibility undergoes a similar enhancement. Since, the imaginary part of the susceptibility is a measure of the spectrum of magnetic excitations, the enhanced RPA expressions

$$\mathrm{Im}[\chi^{+-}(q;\nu)] = \frac{\mu_B^2 \mathrm{Im}\chi_0^{+,-}(q;\omega)}{\left[1 - U\mathrm{Re}\chi_0^{+,-}(q;\nu)\right]^2 + \left[U\mathrm{Im}\chi_0^{+,-}(q;\nu)\right]^2} \tag{10}$$

show the propensity for low-frequency large-amplitude spin-fluctuation excitations, as shown in Fig. 2. Near the instability, the magnetic excitation spectrum consists of a continuum of low-energy (quasi-elastic) and over-damped precritical fluctuations from which, on increasing U above U_c, a branch of sharp spinwave excitations is expected to emerge in the magnetically ordered state. These modes are expected to correspond to the Goldstone modes obtained because the continuous spin rotational symmetry of the Hamiltonian is spontaneously broken in the magnetically ordered state [12].

The above analysis is only approximate in that it only takes into account the first order Gaussian fluctuations about the mean-field approximation. However, since the phase space of a

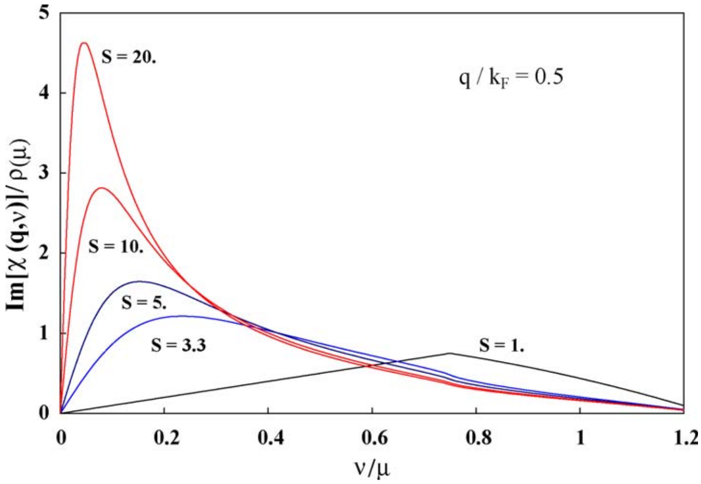

Fig. 2. The spectrum of magnetic excitations as a function of (ν/μ), for various values of the Stoner enhancement factor S, where $S^{-1} = 1 - U\rho(0)$. The spectrum of excitations is given by the imaginary part of $\chi(q,\nu)$. For large enhancements, the spectrum exhibits a high density of low-energy excitations.

quantum system is given by (q,ν), the system is at an effective dimensionality given by

$$d_{\mathrm{eff}} = d + z \tag{11}$$

where z is the dynamical critical exponent. The dynamical critical exponent describes how ν changes as the length scale of the system is changed [10,13,14].

That is, the dynamical critical exponent is defined by the relation between the characteristic frequency ν_c and the correlation length ξ

$$\nu_c \propto \xi^z \tag{12}$$

Since for a ferromagnetic quantum critical point the magnetization is a conserved order parameter, one finds that

$$\mathrm{Im}\chi_0(q;\nu) \propto \frac{\nu}{q} \tag{13}$$

Hence, as

$$\mathrm{Re}\chi_0(q;\nu) \sim \rho(0)\left[1 - a\left(\frac{q}{k_f}\right)^2 + \cdots\right] \tag{14}$$

one finds that for a three-dimensional ferromagnetic quantum critical point $z = 3$. Hence, the effective dimensionality d_{eff} is greater than the upper critical dimensionality, $d_c = 4$. Therefore, the critical fluctuations scale the same way as in the mean-field approximation. This allows one to use the RPA as it provides the correct functional form and scaling description of the quantum critical fluctuations. The RPA also correctly describes the coupling of the magnetic fluctuations to the quasi-particle excitations. In the next section, we shall examine the effects of the paramagnon fluctuations on the quasi-particle excitations and how they become manifest in the de Haas–van Alphen oscillations.

3. The de Haas–van Alphen oscillations

Since we are mainly concerned with the effect of a ferromagnetic quantum critical point, we can restrict our attention to the low electron density limit. In this limit and in the presence of a magnetic field H, a system of electrons will traverse Landau orbitals. The motion of the electrons in the direction parallel to the field is uninfluenced by H, but the motion perpendicular to the field is confined to circular orbitals. The electronic dispersion relation reduces to

$$\varepsilon_\sigma(k_z, l) = \frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c\left(l + \frac{1}{2}\right) + \sigma\mu_B H - \mu \tag{15}$$

where ω_c is the cyclotron frequency

$$\omega_c = \frac{eH}{mc} \tag{16}$$

and m is the electrons band mass. The degeneracy of the level $\varepsilon_\sigma(k_z, l)$ is proportional to $m\omega_c/2\pi\hbar$. Thus, as H increases, the spacing of the Landau levels increases as does their degeneracy. As the occupied Landau levels sweep across the Fermi-energy they become depopulated, thereby leading to an oscillatory term in the thermodynamic potential Ω and hence to an oscillatory part of the magnetization.

In an interacting electron system, the thermodynamic potential Ω is expressed in terms of the electronic self-energy, $\Sigma_\sigma(z)$, by the formula

$$\Omega = \frac{m\omega_c}{2\pi}\sum_{k_z,l,\sigma}\int\frac{dz}{2\pi}f(z)\ln[\varepsilon_\sigma(k_z, l) + \Sigma_\sigma(z) - z] \tag{17}$$

where $f(z)$ is the Fermi–Dirac distribution function. This can be re-written as

$$\Omega = \frac{m\omega_c}{2\pi}\sum_{k_z,l,\sigma}\int\frac{dz}{2\pi}f(z)\tan^{-1}\left[\frac{\mathrm{Im}\Sigma_\sigma(z)}{z - \mathrm{Re}\Sigma_\sigma(z) - \varepsilon_\sigma(k_z, l)}\right] \tag{18}$$

The summation over the Landau orbital index l can be performed with the aid of the Poisson summation formula

$$\sum_{l=0}^{\infty} g(l) = \int_0^\infty dx\, g(x)\left[1 + 2\sum_{p=1}^{\infty}(-1)^p\cos\pi p x\right] \tag{19}$$

which leads to the expression

$$\Omega = -\frac{m\omega_c}{2\pi}\int_0^\infty dx\left[1+2\sum_{p=1}^{\infty}(-1)^p\cos\pi px\right] \times\int_0^\infty\frac{dk_z}{2\pi}\int\frac{dz}{2\pi}f(z)\sum_\sigma\tan^{-1}\left[\frac{\mathrm{Im}\Sigma_\sigma(z)}{z-\mathrm{Re}\Sigma_\sigma(z)-\varepsilon_\sigma(k_z,x)}\right] \tag{20}$$

It is convenient to separate out the spin-dependent terms in the self-energy which results in the exchange splitting. Following the work of Stoner [7], the field-dependence of the mean-field contribution to the self-energy is given by

$$U(n_+-n_-) = 2\mu_B H\; S\; U\rho(0)\left[1+\frac{1}{6}\left(\frac{\rho''}{\rho}\right)-3\left(\frac{\rho''}{\rho}\right)\mu_B^2H^2S^3+\cdots\right] \tag{21}$$

where $S=(1-U\rho(0))^{-1}$ is the Stoner enhancement factor. This shows that the non-linear terms in the filed-dependence of the self-energy cannot be neglected for materials close to a ferromagnetic quantum critical point, specially if the density of states is not constant near the Fermi-energy. In other words, systems close to a ferromagnetic quantum critical point should be expected to exhibit the phenomena of meta-magnetism either if the Fermi-energy is close to a maximum in the density of states or is at an energy where the density of states is rapidly varying. The field-dependence of the exchange splitting is denoted by $\mu_B H\alpha(H)$ where $\alpha(H)$ is given by the expression

$$\alpha(H) = S\left[1+\frac{U\rho(0)}{6}\left[\left(\frac{\rho''}{\rho}\right)-3\left(\frac{\rho'}{\rho}\right)^2\right]\mu_B^2H^2S^3+\cdots\right] \tag{22}$$

which reflects the field-dependence of the experimentally determined spin susceptibility. On separating out the mean-field field-dependence of the self-energy according to

$$\Sigma_\sigma(z) = \Sigma(z)+\sigma\mu_B H(1-\alpha) \tag{23}$$

and then performing the integration over k_z and x, one obtains the expression for the oscillating part of the grand thermodynamic potential

$$\Delta\Omega_{\mathrm{osc}} = -\frac{1}{\sqrt{2\pi}}\left(\frac{m\omega_c}{2\pi\hbar}\right)^{3/2}\sum_{p=1}^{\infty}\frac{(-1)^p}{p^{3/2}}\cos\left[\pi p\left(\frac{m}{m_e}\alpha\right)\right] \times\mathrm{Re}\int_{-\infty}^{\infty}\frac{dz}{2\pi i}f(z)\exp\left[\frac{2\pi p i}{\hbar\omega_c}(z+\mu-\Sigma(z))-i\frac{\pi}{4}\right] \tag{24}$$

in which m_e is the mass of an electron that enters the expression for the Bohr magneton. This equation is similar to that derived by Engelsberg and Simpson [15] for the case of electron–phonon scatterings, and differs only through the insertion of the field-dependent spin splitting factor α. As shown by Onsager [16], the above expression can be generalized to the case of an arbitrary shaped Fermi-surface. In the semi-classical limit, the momentum space area of a

Landau level orbit is quantized and is given by

$$A = \left(l + \frac{1}{2}\right)\frac{2\pi e}{\hbar c}H \tag{25}$$

Therefore, the area enclosed between consecutive Landau tubes is linearly dependent on the field

$$\Delta A = \frac{2\pi e}{\hbar c}H \tag{26}$$

Following Onsager [16], the oscillatory part of the grand thermodynamic potential

$$\Delta\Omega_{\mathrm{osc}} = -\frac{1}{\sqrt{|\ddot{A}_{\mathrm{F}}|}}\left(\frac{m\omega_{\mathrm{c}}}{2\pi\hbar}\right)^{3/2}\sum_{p=1}^{\infty}\frac{(-1)^p}{p^{3/2}}\cos\left[\pi p\left(\frac{m}{m_{\mathrm{e}}}\alpha\right)\right] \times \mathrm{Re}\exp\left[2\pi \mathrm{i}p\frac{A_{\mathrm{F}}}{\Delta A} - \frac{\mathrm{i}\pi}{4}\right]\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{2\pi\mathrm{i}}f(z)\exp\left[\frac{2\pi\mathrm{i}p}{\hbar\omega_{\mathrm{c}}}(z-\Sigma(z))\right] \tag{27}$$

where A_{F} is the extremal are of the Fermi-surface, and $\ddot{A}_{\mathrm{F}}$ is second derivative of the extremal area with respect to k_z. Due to the exponential phase factor of $A_{\mathrm{F}}/\Delta A$, the thermodynamic potential exhibits periodic oscillations as a functions of $1/H$ with period determined by the extremal area of the Fermi-surface $A_{\mathrm{F}.}$ The terms with $p = 1$ yields the fundamental oscillation, while the higher values of p yield the higher order harmonics.

The constant part of the self-energy is observed into the expression for the single-particle energies. The remaining contribution to the electron self-energy is due to the emission and absorption of spin-fluctuations, which is depicted in Fig. 3. The self-energy is usually written as

$$\Sigma(z) = \frac{3}{2}\hbar\sum_q\int_0^{\infty}\frac{\mathrm{d}\nu}{\pi}U^2\mathrm{Im}\chi(q;\nu)\left[\frac{1-f(\varepsilon_{k-q})+N(\hbar\nu)}{z-\varepsilon_{k-q}-\hbar\nu}+\frac{f(\varepsilon_{k-q})+N(\hbar\nu)}{z-\varepsilon_{k-q}+\hbar\nu}\right] \tag{28}$$

where the factor 3/2 incorporates both longitudinal and transverse spin-fluctuations. However, the presence of finite magnetic field breaks the spin-rotational in-variance, and this has important consequences at the ferromagnetic quantum critical point. In the absence of the field and

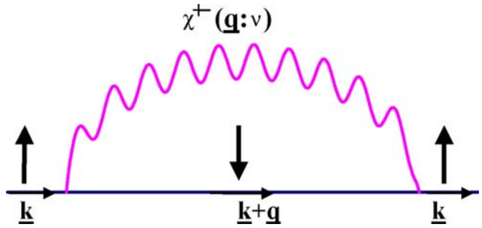

Fig. 3. The electron self-energy due to the emission and absorption of transverse spin-fluctuations. An electron of spin σ emits or absorbs a transverse spin-fluctuation, thereby reversing its spin direction to $-\sigma$.

near zero temperature, the self-energy evaluated at the Matsubara frequencies has the form

$$\mathrm{i}\Sigma(\mathrm{i}\hbar\omega_n) = \frac{9}{2}\hbar\omega_n \ln\left[1 + SU\rho(0)\frac{q_0^2}{12k_\mathrm{F}^2}\right] + O(\omega_n^2 - \pi^2 T^2) \tag{29}$$

where q_0 is the cut-off wave vector. The quadratic terms are purely real and, therefore, when analytically continued back to real frequencies gives the usual $\omega^2 + \pi^2 T^2$ dependence of the quasi-particle scattering rate, expected from a Fermi-liquid. The quasi-particle wave function renormalization Z is given by the coefficient of the term linear in ω_n. It is seen that this reproduces the usual logarithmic enhancement of the quasi-particle mass near a ferromagnetic quantum critical point [8]. If one retains the leading temperature dependent corrections, this results in the mass enhancement factor acquiring the form

$$Z = 1 + \frac{9}{2}\ln\left[1 + \frac{U\rho(0)q_0^2/12k_\mathrm{F}^2}{S^{-1} + (\pi^2/6)k_B^2 T^2 U(\dot{\rho}(0)^2/\rho(0) - \ddot{\rho}(0))}\right] \tag{30}$$

which results in the previously known $T\ln T$ dependence of the electronic specific heat for a system exactly at the quantum critical point. The effect of the finite field will produce a logarithmic reduction of the effective mass and, hence, produce a $T\ln T$ dependence of the specific heat. The applied magnetic field drives the system away from the quantum critical point and thereby stabilizes the Fermi-liquid phase. The difference in the field-dependence of the longitudinal and transverse spin-fluctuations is such that T/H scaling is not to be expected.

On substituting the asymptotic form for the self-energy given by Eq. (29) into the expression of Eq. (27), one finds the Lifshitz-Kosevich expression for the oscillatory part of the thermodynamic potential as

$$\begin{aligned}\Delta\Omega_\mathrm{osc} = {}& \left(\frac{|e|H_z}{2\pi\hbar c}\right)^{3/2} \frac{k_B T}{|\ddot{A}_\mathrm{F}|^{1/2}} \sum_\sigma \sum_p \frac{1}{p^{3/2}} \frac{\exp\left[-\pi p/\omega_\mathrm{c}^* \tau^*\right]}{\sinh 2\pi^2 p k_B T/\hbar\omega_\mathrm{c}^*} \\ & \times (-1)^p \cos\left[\pi p\left(\frac{m}{m_\mathrm{e}}\alpha\right)\right]\cos\left[2\pi p\left(\frac{A_\mathrm{F}}{\Delta A} - \frac{\pi}{4}\right)\right]\end{aligned} \tag{31}$$

in which the cyclotron frequency w_c^* is reduced since it contains the renormalized quasi-particle mass $m^* = Zm$. The enhanced quasi-particle lifetime τ^* is also renormalized via $\tau^* = Z\tau$, where the electron scattering rate is given by the imaginary part of the self-energy

$$\frac{\hbar}{2\tau} = \mathrm{Im}\Sigma(0) \tag{32}$$

As previously mentioned in the discussion of the self-energy, the imaginary part of the self-energy, at the Fermi-surface, is proportional to T^2 as is expected for Barber scattering, and is proportional to a factor of S. Since, the oscillations result from the series of Landau peaks in the electron density of states sweeping across the Fermi-energy and emptying out, thermal broadening due to the finite temperature Fermi-function and finite scattering rates reduce the amplitudes of the oscillations. The effects of the lifetime-broadening of the density of states is represented by the Dingle factor

$$\exp\left[-\frac{\pi p}{\omega_c^* \tau^*}\right] \tag{33}$$

The magnetic fluctuations have the effects of decreasing the amplitude due to the mass-renormalization present in ω_c^* . However, the corresponding factor of Z in ω_c^* in the Dingle term cancels with a similar factor of Z that appears in the quasi-particle lifetime τ^*. As a result, the Dingle term only suppresses the amplitudes of the oscillations with one factor of S present in its exponent. Thermal smearing produces a reduction in the amplitudes and is represented by the thermal reduction factor

$$\left(\sin\frac{2\pi^2 p k_B T}{\hbar\omega_c^*}\right)^{-1} \tag{34}$$

The above analysis has neglected the effect of the frequency variation of the imaginary part of the self-energy. The frequency variation suppresses the contribution of the higher Matsubara frequencies on the oscillations. The leading contribution is given by

$$\exp\left[-\frac{2\pi^2 p k_B T}{\hbar\omega_c^*}\right] \tag{35}$$

and the summation of the higher order terms can only be performed numerically. Since these forms agree at high temperatures, it follows that the effective mass can be determined from the thermal reduction of the oscillations amplitudes.

4. Conclusions

We have examined the effect of magnetic fluctuations on the quasi-particle spectrum, near a quantum critical point. The quasi-particle masses are enhanced by a factor which depends logarithmically on the temperature and the applied field. This analysis reproduces the $T\ln T$ term found in the electronic specific heat and shows that, for large applied fields, the $T\ln T$ variation is replaced by a $T\ln H$ dependence. Therefore, the applied magnetic field stabilizes the Fermi-liquid phase. For finite T and H, there should be no H/T scaling, due to the different field-dependencies of the transverse and longitudinal components of the dynamical magnetic susceptibility. The quasi-particle scattering rate is also expected to be renormalized by the same logarithmic factor. The renormalization of the quasi-particle lifetime should be manifested in optical absorption measurements as a logarithmic narrowing of the width of the Drude peak. All the characteristic properties of the quasi-particle spectrum should be directly observable from measurements of the de Haas–van Alphen oscillations. However, such experiments are only likely to be successful if carried out with large fields since the sheets of the Fermi-surface with heavier masses are difficult to observe.

Acknowledgements

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References

[1] Steglich F, Aaarts J, Bredl CD, Lieke W, Meschede D, Franz W, et al. Superconductivity in the presence of strong pauli paramagnetism: CeCu2Si2. Physical Review Letters 1979;43:1892–6.
[2] Ott HR, Rudigier H, Fisk Z, Smith JL. UBe13: an unconventional actinide superconductor. Physical Review Letters 1983;50:1595–8.
[3] von Löhneysen H. Non fermi liquid behavior in the heavy-fermion system CeCu6-xAux. Journal of Physics Condensed Matter 1996;8:9689–706; Schröder A, Lynn JW, Erwin RW, Lowenhaupt M, von Löhneysen H. Magnetic structure of the heavy fermion alloy CeCu5.5Au0.5. Physica B 1994;47-48:199–200.
[4] Anderson PW. Localized magnetic states in metals. Physical Review 1961;124:41–53.
[5] Hewson AC. The Kondo problem to heavy fermions. Cambridge: Cambridge University Press; 1993.
[6] Saxena SS, Agarwal P, Ahilan K, Grosche FM, Haselwimmer RKW, Steiner MJ, et al. Superconductivity on the border of itinerant-electron ferromagnetism in the heavy-fermion systems UGe_2. Nature 2000;406:587–92.
[7] Stoner EC. Ferromagnetism. Reports on Progress in Physics 1947;11:43–112.
[8] Doniach S, Engelsberg S. Low-temperature properties of nearly ferromagnetic Fermi liquids. Physical Review Letters 1966;17:750–3.
[9] Izuyama T, Kim DJ, Kubo R. Band theoretical interpretation of neutron diffraction phenomena in ferromagnetic metals. Journal of the Physical Society of Japan 1963;18:1025–42.
[10] Moriya T. Spin fluctuations in itinerant magnets. Berlin: Springer Verlag; 1985.
[11] Penn DR. Antiferromagnetism in simple metals. Physical Review 1966;142:350–65.
[12] Goldstone J. Field theories with superconductor solutions. Nuovo Cimento 1961;19:154–64.
[13] Hertz JA. Quantum critical phenomena. Physical Review B 1976;14:65–1184.
[14] Millis AJ. Effect of a nonzero temperature on quantum critical points in itinerant fermion systems. Physical Review B 1993;48:7183–96.
[15] Engelsberg S, Simpson G. Influence of electron-phonon interactions on the de Haas-van Alphen effect. Physical Review B 1970;2:1657–65.
[16] Onsager L. Interpretation of the de Haas-van Alphen effect. Philosophical Magazine 1952;43:1006–8.