Quantum Binary Polyhedral Groups
And Their Actions On Quantum Planes

Chelsea Walton

Joint work with Kenneth Chan, Ellen Kirkman, and James Zhang

November 18, 2012
An investigation of noncommutative/ Hopf invariant theory...
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...quantizations of results in classical invariant theory
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...quantizations of results in classical invariant theory

**Actions of finite subgroups of** $SL_2(\mathbb{C})$

**on**

“planes” $\mathbb{C}[u, v]$
Goal

An investigation of noncommutative/ Hopf invariant theory...
...quantizations of results in classical invariant theory

Actions of quantum finite subgroups of $SL_2(\mathbb{C})$

on

“quantum planes”: noncommutative $\mathbb{C}[u, v]$
Let’s recall some classical results. 

Take $G$ a finite subgroup of $GL_2(k)$ acting faithfully on $k[u, v]$. 

Put $k = \mathbb{C}$
Let’s recall some **classical results**.

Take $G$ a finite subgroup of $GL_2(k)$ acting faithfully on $k[u, v]$.

[STC] $k[u, v]^G$ regular?

\[
k[u, v]^G \cong k[u', v'] \iff G \text{ is generated by reflections.}
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[Klein] Finite subgroups of $SL_2(k)$ are classified up to conjugation.
  
  types: $A_n$ $D_n$ $E_6$ $E_7$ $E_8$

“binary polyhedral groups” =: $G_{BPG}$

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- Types: $A_n$, $D_n$, $E_6$, $E_7$, $E_8$

- “binary polyhedral groups” $=: G_{BPG}$

...they are not generated by reflections.

**[DuVal-McKay]** Geometry of $k[u, v]^{G_{BPG}}$.

The “Kleinian” or “DuVal” singularities $X = \text{Spec}(k[u, v]^{G_{BPG}})$ are precisely the rational double points and the resolution graph of $X$ is Dynkin.
“quantum finite subgroups of $SL_2(k)$” acting on “quantum planes”
Objects of Study

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For $q \in k^\times$, categorically--

- **quantum groups** - dual to - **Hopf algs**

  - $SL_q(2) \cdots \cdots \cdots \mathcal{O}_q(SL_2(k))$
  - $G_q$ fin. subgrp $\cdots \cdots \mathcal{O}_q(G)$ fin. Hopf quot.
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Finite dim’l Hopf algebras $H$

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...that are not necessarily finite quotients of $\mathcal{O}_q(SL_2(k))$

with structure: $(H, m, \Delta, u, \varepsilon, S)$
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AS regular algebras $R$ of gldim 2

AS = Artin-Schelter
* $R$ is graded with $R_0 = k$
* global dimension 2
* AS-Gorenstein
* polynomial growth
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Viewed as ‘noncommutative $k[u, v]$’ in
Noncommutative Projective AG
**Objects of Study**

“quantum finite subgroups of $SL_2(k)$” acting on “quantum planes”

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Classified up to isomorphism:

- $k_q[u, v] := k\langle u, v \rangle / (vu - quv), \ q \in k^\times$
- $k_J[u, v] := k\langle u, v \rangle / (vu - uv - u^2)$
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$k_J[u, v] := k\langle u, v \rangle/(vu - uv - u^2)$

$H$ acts on $R$ if $R$ is a left $H$-module algebra: $R$ is a left $H$-module and

$h \cdot (ab) = \sum (h_1 \cdot a)(h_2 \cdot b)$ and $h \cdot 1_R = \epsilon(h)1_R$ for all $h \in H$, and for all $a, b \in R$
Let $H \neq k$ be a finite dimensional Hopf algebra acting on an AS regular algebra $R$ of global dimension 2.

(H1) [notion of faithfulness]

(H2) $H$ preserves the grading of $R$

(H3) [notion of $H$-action having ‘determinant 1’]

... as results involving $G$ with $\det(G) = 1$ motivate our results. See [DuVal-McKay] for instance.
Let \( H \neq k \) be a finite dimensional Hopf algebra acting on an AS regular algebra \( R \) of global dimension 2.

(H1) \( H \) acts on \( R \) inner faithfully: there is not an induced action of \( H/I \) on \( R \) for any nonzero Hopf ideal \( I \) of \( H \).

(H2) \( H \) preserves the grading of \( R \).

(H3) [notion of \( H \)-action having ‘determinant 1’] ... as results involving \( G \) with det(\( G \)) =1 motivate our results. See [DuVal-McKay] for instance.
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(H3) $H$-action of $R$ have trivial “homological determinant”.
here, $\text{hdet}_HR: H \to k$ and it is trivial if equal to the counit map $\varepsilon$
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(H1) $H$ acts on $R$ inner faithfully: there is not an induced action of $H/I$ on $R$ for any nonzero Hopf ideal $I$ of $H$.

(H2) $H$ preserves the grading of $R$.

(H3) $H$-action of $R$ have trivial “homological determinant”.

**Definition.** A Hopf algebra $H$ satisfying the conditions above is called a quantum binary polyhedral group, denoted by $H_{QBPG}$.
Main Result

**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{AS_{reg2}})\) are classified as follows.
Main Result

**Theorem.** [CKWZ] The pairs $(H_{QBPG}, R_{Asreg2})$ are classified as follows.

**H noncom & s.s.**

$(kG_{BPG}, k[u, v])$

$G_{BPG}$ nonabelian

$(kD_{2n}, k^{−1}[u, v])$

$n \geq 3$

$(\mathcal{D}(G_{BPG})^°, k^{−1}[u, v])$

$\mathcal{D}(G_{BPG})$: Hopf deformation of nonabelian b.p.g. [BN]
**Main Result**

**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{Asreg})\) are classified as follows.

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**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{Asreg2})\) are classified as follows.

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**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{ASreg2})\) are classified as follows.

\[
R = k[u, v] \implies H = kG_{BPG}, \text{ no "new" } H
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**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{ASreg2})\) are classified as follows.

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For \(R = k_q[u, v]\) with \(q\) a root of unity, \(q^2 \neq 1\)

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**Main Result**

**Theorem.** [CKWZ] The pairs $(H_{QBPG}, R_{ASreg2})$ are classified as follows.

For $R = k_q[u, v]$ for $q$ not a root of 1

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For \(R = kJ[u, v]\)

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a finite dimensional Hopf algebra \(H\) acts inner faithfully and preserves the grading of an AS regular algebra \(R\) of gldim 2, with \(H\)-action having trivial homological determinant

we have the following results.

\[
R^H = \{ r \in R \mid h \cdot r = \epsilon(h)r \text{ for all } h \in H \}
\]

[On the regularity of the invariant subring \(R^H\), motivated by [STC]]

[On the Gorenstein condition for the invariant subring \(R^H\), motivated by [Watanabe]]
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**Theorem.** [CKWZ] Let \((H, R)\) be as above with \(H\) semisimple. If \(R^H \neq R\), then \(R^H\) is *not* AS-regular. \((R^H\) has \(\infty\) gldim.)

[On the Gorenstein condition for the invariant subring \(R^H\), motivated by [Watanabe]]
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a finite dimensional Hopf algebra \(H\) acts inner faithfully and preserves the grading of an AS regular algebra \(R\) of gldim 2, with \(H\)-action having trivial homological determinant

we have the following results.

\[
R^H = \{ r \in R \mid h \cdot r = \epsilon(h)r \text{ for all } h \in H \}
\]

**Theorem.** [CKWZ] Let \((H, R)\) be as above with \(H\) semisimple. If \(R^H \neq R\), then \(R^H\) is *not* AS-regular. \((R^H\) has \(\infty\) gldim.)

**Proposition.** [CKWZ] Let \((H, R)\) be as above. The invariant subring \(R^H\) is AS-Gorenstein. (semisimple case by [KKZ])
Future Work

(1) Since $R^H$ is Gorenstein and is not regular ...
Motivated by [DuVal-McKay] and others:

Study the geometry of ‘noncommutative Gorenstein singularities’ $R^H$
for $(H, R)$ in the main theorem, particularly with $H$ semisimple.
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(3) Since AS regular algebras of gldim 3 have been classified...

Study finite dim’l Hopf algebra actions on AS reg. algs of gldim 3.

... AS regular algebras of gldim $> 3$ have not been classified
References:


[DuVal-McKay] P. du Val, On isolated singularities of surfaces which do not affect the conditions of adjunction, 1934; J. McKay, Graphs, singularities, and finite groups, 1980.


[STC] = [Ben93, Theorem 7.2.1]

[Watanabe] = [Ben93, Theorem 4.6.2]

Thank you for listening!