Math 3098: Modern Algebra 1. Instructor: Walton

**Homework #3 on §1.4-1.7 of Goodman.**

Include full statements of problems in your solution set.

Write complete proofs, when proofs are requested.

Don't copy proofs from the internet!

Be honest– show what you know.

Due: Thursday, September 15, 2016 at the beginning of class **sharp**

(1) [2 points] Take an equilateral triangle on the x-y plane with vertex [2] at (0,1). Suppose that all three vertices are equidistant from the origin (0,0). Label the bottom left vertex as [1] and the bottom right vertex as [3].

(a) Find the coordinates of vertices [1] and [3].

(b) Recall that there are six linear isometries (symmetries) of this triangle:

\[ \{e, r_1, r_2, a, b, c\} , \]

where \( e \) is the identity, \( r_1 \) is rotate counter-clockwise by 120 degrees, \( r_2 \) is rotate counter-clockwise by 240 degrees, \( a \) is flip across \( y \)-axis, \( b \) is flip across a line \( y = \beta x \) with \( \beta < 0 \), and \( c \) is flip across a line \( y = \gamma x \) with \( \gamma > 0 \). Find the scalars \( \beta \) and \( \gamma \).

(c) Now complete the table

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as discussed in the September 6th lecture for a square.

(2) [1 point] Prove that two disjoint cycles of \( S_n \) commute, for \( n \geq 4 \), in your own words. (Can use ideas from September 6th lecture.)

(3) [1 point] Exercise 1.5.10 from Goodman.

(4) [1 point] Exercise 1.6.6 from Goodman.

(5) [1 point] Exercise 1.6.8 from Goodman.

(6) [1 point] Exercise 1.6.9 from Goodman.

(7) [1 point] Exercise 1.7.4 from Goodman.

(8) [1 point] Exercises 1.7.11 and 1.7.12 from Goodman.

(9) [1 point] Exercise 1.7.16 from Goodman (be sure to show work).