Corrections and Updates for "Multiplicative Invariant Theory"

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10/15/2019

page 18, line 6: G and H are interchanged. It should read "Then the G-module $\mathbb{Z}[G] \otimes_{\mathbb{Z}[H]} M$ is in fact a G-lattice; ..." (Thanks to Robert Fossum.)

page 21, line -1: A backslash is missing in front of "emph". It should be "The group G is called k-*reflection group* on V if $G = \mathcal{R}^k_V(G)$." (Thanks to Robert Fossum.)

page 60, Table 3.1: The Hironaka decomposition of the invariant algebra $\mathbb{Z}[L]^{\mathcal{G}}$ for the group $\mathcal{G} \cong \mathcal{S}_3$ (third group in table) is missing the summand $\mu_3^2\, \mathbb{Z}[\mu_1, \mu_2]$. The corrrect entry in this row should be: "semigroup algebra $\mathbb{Z}[\mu_1, \mu_2] \oplus \mu_3\, \mathbb{Z}[\mu_1, \mu_2] \oplus \mu_3^2\, \mathbb{Z}[\mu_1, \mu_2]$".

page 103, line -1: "k-reflection" should be replaced by "bireflection"; so

$$\mathcal{R}^2(\mathcal{H}) = \langle g \in \mathcal{H} \mid g \text{ acts as a bireflection on } L \rangle \ .$$

page 126, line 8 and page 127, line 4: The list of "primes" $p = 47, 112, 223, \dots$ is wrong as stated; it should be $p = 47, 113, 233, \dots$. Thus, on page 127, it should read "In particular, the aforementioned non-rational extensions $\mathbb{Q}(\mathrm{Cl}_p)/\mathbb{Q}$ ($p = 47, 113, 233, \dots$) are not stably rational either." (Thanks to Don Passman.)

page 126, lines -9 and -12: K should be replaced by $\Bbbk$ twice: it should read " ...Saltman [175] for infinite $\Bbbk$ ..." and " ... E/F of $\Bbbk$ with group $\mathcal{G}$ ...."

page 149, line -4: The answer to Problem 2, when stated more generally for k-reflections, is definitely negative. Alex Zalesskiĭ has shown me the following example. The simple group $\mathrm{SL}_2(\mathbb{F}_{32})$ has an irreducible Steinberg module, V , of dimension 32 over $\mathbb{Q}$. The restriction of V to the Sylow 2-subgroup of $\mathrm{SL}_2(\mathbb{F}_{32})$ is the regular module. Therefore, each involution of $\mathrm{SL}_2(\mathbb{F}_{32})$ acts as a 16-reflection. Furthermore, the involutions generate $\mathrm{SL}_2(\mathbb{F}_{32})$. On the other hand, if $g \in \mathrm{SL}_2(\mathbb{F}_{32})$ is an element of order 31, then its fixed points on V have dimension 2 and $\langle g \rangle$ is an isotropy group on V. Thus, this isotropy group is only generated by a 30-reflection.

page 150, line 12: Omit "over $\mathbb{Z}$".

page 154, line 5 after Problem 7: Replace $|\mathcal{G}|/r$ by $r/|\mathcal{G}|$ (twice).

Problem 14 (page 159): This has been solved in the affirmative; the reference is:

> A. Hoshi and Y. Rikuna, *Rationality problem of three-dimensional purely monomial group actions: the last case*, Math. Comp. **77** (2008), 1823–1829.

Now all multiplicative invariant fields of transcendence degree at most 3 are known to be rational over the base field.

Example 8.11.3 (page 122); see also Problem 4 (page 150/1): The Cohen-Macaulay problem for "vector invariants" is resolved (in the positive): the Cohen-Macaulay property follows as a special case of Theorem 1.2 in

> B. Blum-Smith and S. Marques, *When are permutation invariants Cohen-Macaulay over all fields?*, Algebra & Number Theory **12** (2018), 1787–1821.