1. Compute $2^{2r} \pmod{3}$, for all positive integers $r$.

2. Prove that $([a] + [b])^2 = [a]^2 + [b]^2$, for all $[a], [b] \in \mathbb{Z}_2$.

3. Let $t$ be an integer, and let $n$ be a positive integer. Prove that
   
   $\mathbb{Z}_n := \{[0], [1], \ldots, [n-1]\} = \{[t], [t+1], \ldots, [t+(n-1)]\}$.

   For the remainder let $n$ be a positive integer. When the context is clear we refer to $[0]$ in $\mathbb{Z}_n$ simply as zero, and we say (no surprise) that an element of $\mathbb{Z}_n$ not equal to $[0]$ is nonzero. If $r$ is an integer such that $[r]$ in $\mathbb{Z}_n$ is nonzero, then we say that $[r]$ is a zero divisor in $\mathbb{Z}_n$ provided there exists an integer $s$ such that $[s]$ is nonzero and $[r][s] = [s][r] = [0]$, and we say that $[r]$ is invertible in $\mathbb{Z}_n$ provided there exists an integer $u$ such that $[r][u] = [u][r] = [1]$.

4. Let $r$ be an integer. Prove that $[r]$ in $\mathbb{Z}_n$ cannot be both a zero divisor and invertible.

5. For this problem, recall from class or the text the proposition stating that two nonzero integers $a$ and $b$ are relatively prime if and only if there exist integers $u$ and $v$ such that $au + bv = 1$. Now let $r$ be an integer such that $[r]$ is nonzero in $\mathbb{Z}_n$. Prove that $[r]$ is invertible in $\mathbb{Z}_n$ if and only if $r$ and $n$ are relatively prime.