Review of elementary mathematical reasoning and the writing of mathematical proofs.

1. Write the negations of the following statements, avoiding use of the word “not.”
   (a) “The integer \( n \) is odd and the integer \( 3n \) is odd.”
   (b) “The integer \( n \) is odd only if \( 3n \) is odd.”

2. Consider the following statement: “For all real numbers \( x \), there exists a positive integer \( N \) such that \( N - 1 \leq x^2 \leq N \).” Write the negation of this statement, avoiding use of the word “not.” (Notice that \( N - 1 \leq x^2 \leq N \) means \( N - 1 \leq x^2 \) AND \( x^2 \leq N \).

3. Let \( m \) be an integer. Prove: If \( m \) is odd then \( m^2 + 1 \) is even. (Note: For this homework assignment, your proofs should only use elementary integer arithmetic and the definitions that \( m \) is odd when \( m = 2k + 1 \) for some integer \( k \) and that \( m \) is even when \( m = 2\ell \) for some integer \( \ell \).)

4. Let \( m \) be an integer. Prove: If \( m^2 + 1 \) is odd then \( m \) is even.

5. Let \( m \) and \( n \) be integers, and suppose that \( m^2 + n^2 \) is odd. Prove that at least one of \( m \) or \( n \) must be odd.

6. Give a complete proof by induction that
   \[
   \sum_{i=1}^{n}(2i) = 2 + 4 + 6 + \cdots + 2n = n^2 + n
   \]
   for all positive integers \( n \). Make sure to clearly indicate your basis step and induction step.

7. Give a complete proof by induction that \( 4^n > n^2 \) for all non-negative integers \( n \). Make sure to clearly indicate your basis step and induction step.