1. Let $A$ and $X$ be sets, $\mathcal{F}$ the collection of all functions from subsets of $A$ to $X$. Define a partial order on $\mathcal{F}$ as follows: if $f : B \to X$, $f' : B' \to X$, then $f \preceq f'$ iff $B \subset B'$ and $f'|_B = f$. Let $\mathcal{C}$ be a totally ordered subset of $\mathcal{F}$. Show that $\mathcal{C}$ has an upper bound in $\mathcal{F}$: there is $g \in \mathcal{F}$ such that $f \preceq g$ for all $f \in \mathcal{F}$. (Here $g$ need not belong to $\mathcal{C}$.)

2. Let $X$ be a topological space, suppose is the union of a family of compact spaces $\{X_\alpha\}_{\alpha \in A}$: $X = \bigcup_{\alpha \in A} X_\alpha$. Let $\mathcal{F}$ be the collection of all maps $A \to X$ such that $f(\alpha) \in X_\alpha$. Let $\mathcal{N} = \{f_d\}_{d \in D}$ be a net in $\mathcal{F}$. Show that there is $f \in \mathcal{F}$ and a subnet $\mathcal{N}' = \{f_{d'}\}_{d' \in D'}$ of $\mathcal{N}$ such that for each $\alpha$, the net $\{f_{d'}(\alpha)\}_{d' \in D'}$ converges to $f(\alpha)$. Hint: Analyze the proof of Tychonoff’s theorem.