Problem Set #6, due Nov. 19.

1. Let \( \{a_{n,m}\}_{m,n \in \mathbb{Z}} \subset \mathbb{C} \) be such that
   \[\sum_{m,n=-\infty}^{\infty} |a_{m,n}| < \infty.\]

   Show that the limit
   \[S = \lim_{N \to \infty} \sum_{m^2+n^2 < N} a_{n,m}\]
   exists.

2. With the setup of Problem 1, show that
   \[S = \lim_{N \to \infty} \lim_{M \to \infty} \sum_{n=-N}^{N} \sum_{m=-M}^{M} a_{n,m}\]

3. Let \((X, \mathcal{M}, \mu)\) be a measure space and \(\nu\) a signed measure on \(\mathcal{M}\). Suppose \(\nu \ll \mu\). Show that \(\nu^- \ll \mu\) and \(\nu^+ \ll \mu\).

4. Let \(\{r_n\}_{n=1}^{\infty}\) be an enumeration of the rational numbers, let \(\{c_n\}_{n=1}^{\infty}\) be a sequence of nonzero real numbers such that \(\sum |c_n|\) converges. Define \(\nu : \mathcal{B} \to \mathbb{R}\) by
   \[\nu(E) = \sum_{n : r_n \in E} c_n\]

   Show that \(\nu\) is a signed measure, describe all Hahn decompositions of \(\mathbb{R}\) with respect to \(\nu\), and find the Jordan decomposition of \(\nu\).

5. Let \(\mu : \mathcal{B} \to [0, \infty]\) be defined by
   \[\mu(E) = \#\{E \cap \mathbb{Q}\}\]

   Let \(\nu\) be the measure of Problem 4. Show that \(\nu \ll \mu\).