1. Let $X, \mathcal{M}, \mu$ be measure space. Suppose that there is a sequence $\{E_n\}$ of measurable sets of finite measure bounded below by some positive number with the property that if $F \in \mathcal{M}$ and $\mu(F) < \infty$ then $\mu(F \cap E_n) \to 0$ as $n \to \infty$. Find a sequence of elements $\phi_n \in L^+(X, \mathcal{M})$ with $\int \phi_n = 1$ but $\phi_n \to 0$ pointwise a.e.

2. Let $X, \mathcal{M}, \mu$ be a measure space. Suppose there is a sequence of measurable sets $F_n, n = 1, 2, \ldots$, with $F_n \supset F_{n+1}$ of positive measure $\mu(F_n) \to 0$ as $n \to \infty$. Find a sequence of elements $\phi_n \in L^+(X, \mathcal{M})$ with $\int \phi_n = 1$ but $\phi_n \to 0$ pointwise a.e.

3. Let $X, \mathcal{M}, \mu$ be a measure space. Suppose $f, g$ and $f_n, n \in \mathbb{N}$, are measurable functions $X \to \mathbb{C}$ with $g \in L^1, |f_n| \leq g$ and $f_n \to f$ in measure. Show that $f \in L^1$ and $\lim \int f_n = \int f$.

4. With the setup of the previous problem, show that $f_n \to f$ in $L^1$.

5. Let $f_n : [0, 1] \to \mathbb{R}$ be defined by $f_n(x) = x^n, n = 1, 2, \ldots$. Show:
   (1) $f_n \to 0$ pointwise a.e.
   (2) $f_n \to 0$ in measure.
   (3) $f_n \to 0$ in $L^2$.

6. Let $X, \mathcal{M}$ be a measurable space, $Y, d$ a metric space, and $f, g : X \to Y$ $\mathcal{M}$-$\mathcal{B}$ measurable functions. Show that
   $X \ni x \mapsto d(f(x), g(x)) \in \mathbb{R}$
is $\mathcal{M}$-$\mathcal{B}$ measurable.