Topic 3. Problems motivated by geography

3.1. Assume that the Earth's orbit is circular, the year is 365 days long and the axial tilt is 23.5 degrees. Find a formula (as a function of time) for the latitude of the point on Earth where at local noon the Sun's light falls perpendicularly to the Earth at that point. Also find the approximate days of the year where at midday the light falls perpendicularly on the parallel at 17.043° north.

▲ The time between successive vernal equinoxes (around March 20 each year in the northern hemisphere) is 365.25 days (365.24219 is more precise); assume it occurs at midnight (from the 20th to the 21st) in your longitude. The Earth's axial tilt is the angle between the axis of rotation of our planet and the perpendicular to the ecliptic, the plane of its orbit around the Sun, approximately 23.5 degrees (23.437°). On the vernal equinox, the projection of the Earth's axis of rotation on the ecliptic is tangent to the orbit. Assume the the Sun's light rays arrive parallel to each other and the Earth orbital speed is constant,

3.2. Assume the Earth is a ball of perimeter $40,000$ kilometers. There is a building 20 meters tall at point a. A robot with a camera placed at 1.75 m. above the surface of the Earth starts walking away from the building. At what distance from a, computed on the surface, does the robot cease to see the (top of the) building?

3.3. The point with latitude and longitude 48.8567°N, 2.3508°E, respectively, lies somewhere in Paris, while that corresponding to 39.9500°N, 75.1667°W lies in Philadelphia (data obtained through Google). Assuming the surface of the Earth is a sphere of circumference $40,000$ kilometers, estimate the distance between Philadelphia and Paris.

3.4. Assume that the orbit of the Earth around the Sun is a circle with radius $149,597,870,700$ kilometers. Assume that the Sun is a point and the Earth is a sphere of radius $40,000/2\pi$ kilometers that rotates once around its axis in 24 hours, and that the axis is tilted 23.4° with respect to the perpendicular to the plane of the orbit (the direction of the axis does not change as the Earth moves around the Sun). Compute the maximum and minimum daylight duration at the latitude of Philadelphia (use 39.9500°N) in hours and minutes.[1] You may approximate to the nearest minute

3.5. Assume the Sun is very far away from Earth so that light rays arrive parallel to each other. The task is to determine Earth's circumference, assuming that it is spherical, using the following information. There are two 10 meter poles placed vertically (normally to the surface) on the same meridian, both on the northern hemisphere, at distance 790 km from each other. At noon on the same day one of the poles casts a shadow (towards the north) measuring 11.3 cm, the other, 136 cm (also towards the north).

3.6. Calculate the time between the moment in the morning when sunlight first shines on a camera placed on the surface of the Earth at the equator and the time in the evening when sunlight ceases to illuminate the camera. Make the same

[1] As in 10 hours 59 minutes. All values of physical constants here and below were taken from wikipedia

assumptions as above; in addition, assume the sun subtends an angle of $0.53°$ as seen from Earth and a day is 24 h long.

3.7. Again suppose the Earth is a ball with circumference 40,000 km. Suppose a very small satellite has a circular orbit in the plane of Earth's equator, at distance 385,000 km from the center of the Earth with period 29.5 d. The Earth is illuminated by a very far away point source of light (the Sun, idealized) and the satellite casts a shadow on Earth as it passes between the light source and the Earth. Write a formula for the speed of the shadow as it traverses the equator on the day of the vernal equinox, assuming the satellite moves in the same direction as the rotation of the Earth. Take into account that a day is 24 h long.

3.8. Taking Newton's gravitational constant to be

$$G = 6.67408 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2},$$

Earth's mass as

$$m_{\mathrm{E}} = 5.972 \times 10^{24}\,\mathrm{kg},$$

and a day to be 24h long, compute the radius of the orbit of a satellite that "remains" over the same spot of the equator throughout the day. More generally, find the radius of the orbit of a satellite moving in circular orbit in a plane through Earth's center with normal making angle $0 < \alpha < \pi/2\,\mathrm{rad}$ with Earth's axis of rotation, under the requirement that the satellite crosses the equatorial plane every 12 h.