1. Introduction

This portfolio contains a record of my growth as a teacher while at Temple University from Fall 2017 to Spring 2019. Following my statement of teaching practice are vignettes of my courses. Each includes a description of the specific methods from my practice that I employed and reflections on how the course went, a syllabus excerpt for non-coordinated courses (excluding university required language), a sample classroom activity or assessment, and course evaluations.

2. Statement of Teaching Practice

The central goal of my teaching practice, at all levels of instruction, is to aid students in discovering their identity as a mathematical thinker. The specific growth I seek to nurture varies depending on the level of the student, but at all levels I work towards this goal by using group instruction to foster a classroom community where all students feel welcome to experiment with new expressions of their mathematical identity. Finding community is only part of building an identity—equally important is developing a personal voice—and I use writing extensively, as an instructional exercise and as formative and summative assessment, to give my students ample opportunity to build their mathematical voice.

My students, both in Philadelphia and in Chicago, have come from diverse racial, ethnic, and economic backgrounds. Helping students identify as mathematical thinkers has a well-documented positive impact in diverse classrooms. The positive outcomes I have obtained in these classrooms has led me to center this goal. In approaching this goal I have taken advantage of the small class sizes available at Temple, where most courses have at most 36 seats. I have been guided by several professional development activities which have cultivated specific strategies well-suited to a small class setting.

To build a community where my students feel welcome I structure my classroom around group activities and social interaction. I was first introduced to these teaching methods as a graduate student, leading workshops based on Uri Triesman’s Berkeley model. In these workshops I took on the role of guide, leading the student groups through challenging problems with leading questions and soliciting student presentations of partial ideas (including ideas that led to dead ends). Through participating in MAA’s Project NExT I further developed the teaching methods I learned in Chicago.
Building on this training and experience I used a community centered approach in my teaching at Temple. My service courses were taught in a flipped style. Prompted by a Project NExT workshop I included tactile manipulatives in the in-class group activities. One of the most successful tactile lessons was in a multivariable calculus course. After reading about parametric curves before class students used wire to construct a graph of a curve in space with their group. Once groups were happy with their constructions, the whole class circulated among the graphs in a matching activity which generated a dynamic discussion of the interactions between the three coordinate expressions and the resulting shape. This lesson was a favorite of the students, but I was most pleased with the ownership students took of their graphs—taking them home to show to friends, turn into jewelry, or decorate their rooms. These actions are the seeds of a mathematical identity and for the remainder of the unit on curves students readily turned to the particular parametric curves they had modeled when I suggested they try out a new concept on a familiar example.

In smaller upper-division courses it is easier to allow student discovery and ownership to lead the class; I used a completely inquiry based (IBL) approach in Topology I in Spring 2019. In an IBL classroom students work in small groups to develop the material of the course based on worksheets of leading questions and limited definitional exposition from the instructor. Content goals are formatively assessed regularly by student-to-student presentations of proofs, examples, and counter-examples; as well as by listening to student discussion. I had wanted to try teaching an IBL course since learning about the method through Project NExT—the classroom model strongly supports my central goal. The course succeeded in building a strong sense of community, in the words of one of the students “[this class] has been the only one to expose us to what collaboration looks like in our field ... our class came to appreciate ... the many things we learned from each other”, and in the context of this community each student came to express a sophisticated mathematical identity that emphasized their individual strengths, taste, and background.

Equally important for forming identity is developing a personal voice, and in my experience there is no better tool for this task than writing. Group interactions and presentations certainly help but the writing process, from reflection and brainstorming through revision and polishing, has no equal. In all of my courses I use some form of pre-class written reflection in conjunction with the reading or preparatory activity. These reflections are primarily writing to learn activities, giving students the opportunity to synthesize before entering class, but I also use them as a formative assessment, and shape a day’s lesson based on student responses. Conic sections were used as a running example in my modern geometry course, and early on in the class the students read the text’s classification of Euclidean conics. The classification was presented algebraically, and in the reflection question I asked the students to give a geometric description of the proof in terms of moving the conic. The responses identified the step in the proof where the conic is translated without an issue but revealed that no one in the class was clear on the difference between diagonalization and orthogonal diagonalization. Based on these responses, I revised my lesson plan to start the day with several visualizations of geometric interpretations of 2 by 2 real matrices; the remainder of the lesson plan expected that students be comfortable with these interpretations.

Like the use of community centered instruction, my use of writing has been influenced by professional development activities. In addition to Project NExT, where I organized a panel on writing in mathematics for my class of fellows, I participated in a faculty learning community at Temple on the delivery of a writing intensive course. A large writing project can form a turning point in the growth of a student’s mathematical identity, as illustrated by Olivia, a student in my Modern Geometry course. In the course I used an extensive research project as a large portion of the summative assessment, replacing a written final exam. Olivia’s project covered the classification of frieze symmetries; in designing a model to accompany her paper Olivia additionally successfully classified which of the frieze symmetries could be realized by a monotile. This was her first encounter with abstract group theory as well as mathematical research, but it transformed her thinking about career paths. Before the course, as a student in our secondary education program, Olivia had not
seriously considered that careers existed in mathematical research and that she could pursue them. The semester after completing the project she came to my office to ask generally about graduate school and specifically about working with me as a summer research student in group theory to gain a deeper understanding of what research felt like. Without the experience of the course project, I do not think Olivia would have added researcher to her mathematical identity so easily. This is not an isolated example. In every course where I have used written assessments student feedback is enthusiastic and the commentary reflects the growth of identity engendered by the writing process.

This fall I am incorporating these methods into a large (160 seat) general education course. The Project NExT workshops did not shy away from the difficulty presented by this task and I feel well-equipped to continue teaching in my preferred style in this new setting, with some modifications. In preparing for this class I have placed a premium on ensuring my lesson designs have many entry points, to give students with diverse backgrounds and goals a variety of opportunities to relate to the course material and develop their own identities. Throughout delivering this course I will be consulting with professional mentors to address challenges faced in scaling up my teaching methods.

Throughout my teaching practice I am and will remain committed to guiding diverse student populations at all levels to discover their mathematical selves as a part of an inclusive mathematical community where they are welcome to express that identity in their personal voice.

3. MATH 2043: CALCULUS III, FALL 2017

MATH 2043 is a coordinated, multi-section course. As a course instructor I was responsible for classroom instruction and setting weekly summative assessments in the form of quizzes. The course content, syllabus, homework, and major assessments were set by the department coordinator.

This course was the first course I taught as instructor of record. A typical 70 minute class period contained 10-20 minutes of conceptual exposition on my part with the remainder of the time spent on group or class wide active work to explore the topic of the day.

Reflecting on this course at the end of the semester, I felt that while the group instruction had been successful, I spent many of my interactions with small groups reiterating some portion of my presentation, instead of building deeper understanding or synthesis. After talking with NExT fellows about the experience during the 2018 Joint Meetings I concluded that I was not giving students enough time to digest the material before asking them more difficult questions about it. As a result, in subsequent classes I incorporated pre-class readings and moved to a completely flipped style of instruction in coordinated courses. In my Precalculus course in the Spring of 2019 this completely flipped style produced noticeably higher quality in-class discussion and engagement.

3.1. Sample activities. These are three separate in-class activity prompts used in the course. They have been re-formatted from the original worksheet layouts for space reasons. The partial derivative activity is based on the corresponding activity in Active Calculus (Austin, Boelkins, and Schlicker, 2017).

Graphing Parametric Curves. For the curve assigned to your group, graph projections to the different co-ordinate planes. Use these plots to build a 3D model with florist’s wire.

\[ r(t) = (\cos(t), \sin(t), t) \]
\[ r(t) = (t, t^2, t^3) \]
\[ r(t) = (\cos(t), \sin(t), \cos(2t)) \]
\[ r(t) = (t \cos(t), t \sin(t), -t) \]

Once each group has finished building their model, circulate through the room and match the models to the other equations. From the models, graph projections of each curve to the different co-ordinate planes.
**Interpreting Partial Derivatives.** The speed of sound through any medium is determined by the density. In the ocean, density is primarily a function of temperature, depth, and salt concentration (salinity). Geophysicists tell us that the function relating the speed of sound and these three quantities is modeled by

\[ C(T, S, D) = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.016D \]

The units are \( C \) the speed of sound in \( m/s \), \( T \) the temperature in \( ^\circ C \), \( S \) is salinity in \( g/L \) grams of salt per liter, and \( D \) is the depth below sea level in \( m \).

1. State the units for each of the partial derivatives \( C_T, C_S, C_D \), and explain the physical meaning of each.
2. Find the partial derivatives \( C_T, C_S, \) and \( C_D \)

\[ C_T(T, S, D) = \quad C_S(T, S, D) = \quad C_D(T, S, D) = \]

3. Evaluate the partial derivatives at the point where \( T = 10^\circ C, S = 35g/L, \) and \( D = 100m \).
4. What does the each partial derivative tell us about the speed of sound if we were to go 100m deep off the coast of Atlantic City (the temperature and salinity are typical averages for the Atlantic ocean in mid-latitudes)? Write your answer in paragraph form, as if you’re making a report to a less technical consulting client.

**Describing Regions in New Coordinates.** Your group will be given a description of a 3D region as a sentence and a 3D plot. You will need to convert this region to a description in terms of inequalities in our different coordinate systems, suitable for use as the limits of an integral.

1. Describe the region with inequalities in rectangular coordinates. Or explain why this is a bad idea.
2. Describe the region with inequalities in cylindrical coordinates. Or explain why this is a bad idea.
3. Describe the region with inequalities in spherical coordinates. Or explain why this is a bad idea.
4. Look at your three different expressions, and set up a triple integral to find the volume of your group’s region that looks easiest to evaluate.
5. Evaluate your integral.
6. Prepare a poster on large chart paper illustrating your group’s thought process and volume calculation.

Once your group has finished your poster hang it on the wall. Circulate around the room reading the posters, and use post-it notes to leave any comments, questions or feedback. Concentrate on the reasons each group had for making their choice of coordinates. Would you have made a different choice? I will lead a discussion of the commentary once everyone has had a chance to view the posters.

3.2. **Selected feedback quotes.**

“The active learning environment was GREAT. I felt very comfortable to ask questions and to participate which resulted in a much deeper understanding of concepts. Working in table groups was super effective. I also appreciated doing class examples with the professor and then doing work with our groups. Overall, one of the best courses I have ever taken. Dr. Bering was definitely an impactful professor. He never made me feel stupid for not understanding something or for getting answers wrong.”

“Dr. Bering was able to demonstrate the application of the equations throughout the course with great use of visual media and group work. The atmosphere is
created in class lead to an enjoyable math learning experience. The group discussions/worksheets were helpful by making me vocalize my thought process allowing me know whether or not I was on the right track in terms of my understanding of the material. His explanations while doing work on the board was simple, understandable, and relatable to work assigned. This was my most enjoyable math course I have taken because of the teaching methods Dr. Bering used.”

“I think the collaborative learning environment was much different than most students are used to, and it took sometime to get into. I feel like our class really got into it the last half of the semester and really helped me learn by feeling comfortable asking questions and being able for the first time to ask why. This is the most comfortable I have ever felt in the Math class and I think that is great.”

4. MATH 9072: Differential Topology II, Spring 2018

MATH 9072 is a graduate level topics course in differential topology. As course instructor I had complete freedom to select course topics. While formal enrollment happened after I selected a topic, I was aware of my likely students and had some informal conversations with them about their interests and goals in our graduate program. The input I received was incorporated into the design of the course.

The course content was delivered via a mix of lecture and in class discussion of exercises, with the majority of class time spent on lectures. Unlike in an undergraduate setting where a single textbook is usually available, this course drew upon a variety of primary sources. The purpose of my lectures was to provide a synthesis of these varied sources into a single common notation and lexicon, highlight the unifying themes, and fill in gaps in proofs or provide alternative presentations. The students were not passive participants in this role; before class they were assigned readings from the primary sources underpinning the content and asked to reflect on them. Their reflections guided my preparation, and at times I yielded the chalk to a student who had suggested a clarifying perspective or interesting tangent in their reflection.

Following the lecture, the students were assigned the task of preparing a course blog that offered a rough write-up of the lecture’s contents. The write-up was expected to connect the lecture notes with the primary sources, offer elaboration the author thought was necessary, and extract exercises from the material to form the basis of the course’s homework problem set. I assigned the students rotating authorship and editorship roles; the student editor was expected to edit for the author, and I had set up a system to observe the editing feedback process. The initial authoring of the blog posts was very valuable for the students, but the editing process became more adversarial than collaborative. In a future course that uses a blog I plan to use co-authorship instead of the author/editor roles using a collaborative writing tool so that I can view my students’ interactions and coach them on the different approaches to mathematical collaboration.

4.1. Syllabus.

Course Description: Before stating his famous conjecture Poincaré claimed, in the second supplement to Analysis situs, that a 3-manifold has the homology groups of the 3-sphere if and only if it is homeomorphic to the 3-sphere. Later, in the fifth supplement, and after almost decade of correspondence with Heegaard and other early topologists, Poincaré discovered a counterexample: his famous homology sphere, a 3-manifold that is not homeomorphic to the 3-sphere but has the same homology groups. This discovery led to the well-known the Poincaré conjecture: a 3-manifold with trivial fundamental group is homeomorphic to the 3-sphere. Since then, the Poincaré homology sphere has been a driving example in low-dimensional topology. In this course we will sample themes (constructions, classification theorems, and structure theorems) from the past century and a quarter of low-dimensional topology, using the Poincaré homology sphere as our guide. We will touch on
and connect knot theory, mapping class groups of surfaces, crystallographic groups, singularities of algebraic surfaces, and more; meeting a menagerie of mesmerizing manifolds in the meanwhile. Our rough list of topics:

(1) Surgery on knots,
(2) Branched covers of knots,
(3) Seifert fibered spaces,
(4) Plumblings,
(5) Surgery on links,
(6) Singularities of algebraic surfaces,
(7) Heegaard splittings,
(8) Group actions on the 3-sphere,
(9) Polyhedral identification spaces.

Readings: I will share a google calendar of the readings with you. The expectation is that readings will be completed before class, especially in cases where several alternative expositions are developed and we will only look at one in class. For some readings I will also ask a reflection question to be answered in the linked google form.

Course Blog: There is a course blog. This will be used to accumulate the course notes, written by the students, revised and commented on by all of us. Blog entries for a given lesson should be based upon the lesson’s content but (as appropriate) fill in the details and references, and add new ones as appropriate. All course members will be added as authors on the site using their Temple IDs.

Exercises: I will mention some of these in lecture; the blog-writer of the day should transcribe them. If you notice particularly good exercises in the course of your reading you’re welcome to suggest them, either by just adding them to your blog-post if you’re the blog writer, or adding them in the comments. At the half-way point in the semester and at the end I’ll collect some write-ups from you for a reasonable number of exercises. Reasonable number will depend on how many exercises are generated.

Presentation: The last week or two of class will be devoted to student presentations. The presentation should be an application or facet of the Poincaré sphere not covered so far in the class. To do this, you will need to either locate a paper on a topic about the Poincaré sphere and present on it, or discover something new on your own. In either case, you should pick your topic and clear it (and the source) with me in the fourth to last week (Week 13). If you are presenting an existing paper, please provide a suggested reading (e.g. it may not be necessary to read the entire paper, just certain points). If you plan on discovering something new, you should have a draft of your proof ready to share with your classmates in advance of your presentation, though it can lack in background and introductory material.

4.2. **Student evaluations.** The Temple electronic student feedback forms are only used in courses with an enrollment of five or more. Instead, for this course, the director of graduate studies conducted an informal non-anonymous student evaluation of teaching. Three of the four students submitted replies, which follow.

**Student 1:**

(1) *What aspects of the instructor’s approach contributed most to your learning in the course?*

Writing the blog posts has been helpful for me since it forced me to take a look back at my note and reinforced my understanding of the materials. The instructor was careful and clear in presenting proofs and was good in supplying the proofs with details (and some time coordinates crunching).
What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?

It may be a good idea to have some homework problems submitted regularly. Throughout the course, we have learned many different constructions of the Poincare homology sphere. But, it would be good to mention to what extend the constructions generalize to give interesting manifolds. Some topics where this can be done (and that I can think of right now) are the link of singularities and central extension of orbifold group.

What aspects of the course contributed to your intellectual or professional development?

It was good that I had an opportunity to get exposed to a large number of topics in low dimensional topics. The final presentation at the end of the course was helpful since it gave me a chance to present in a friendly environment.

Student 2: I combined 1 and 3 because I found it hard to separate into two sections.

What aspects of the instructor’s approach contributed most to your learning in the course?

This course covered a wide breadth of topics sampling of key ideas across several subareas in low-dimensional geometry and topology that I had very little exposure to from my own research. This survey gave me perspective that allows me to better appreciate and understand work by my peers. Moreover, the background I gained from this class has aided me in running a seminar directed at exposing graduate students to fundamental concepts in geometry and topology. The notion of maintaining a course blog was novel to me. The framework that we worked on in this course gave me an sense of alternate means of disseminating understanding and interpretations of ideas within the research community beyond publications in journals. What’s more, though the platform for posting entries certainly had a learning curve, it was approachable and gave me a sense that I am capable of using a similar platform to circulate ideas regarding my own research.

What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?

Boardwork was quite unclear at times. Clearer guidelines on the blog posts. For example, numbering for results and macros. Somehow building in more accountability with homework would have helped. Pacing for the course material could have been augmented to better fit with students’ understanding.

Student 3:

What aspects of the instructor’s approach contributed most to your learning in the course?

Edgar was obviously invested in the students’ learning. He had a clear vision of how to course should proceed, combining lectures with the goal of constructing a course-blog to compile notes and ideas since there was no single source of a detailed exploration of the Poincare homology sphere. The course was approached informally and made a welcome environment.

What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?

The instructor should write on the board more. It was difficult to organize the course-blog because lectures were not clearly structured and propositions were unnumbered (makes cross-referencing to other folks’ notes hard). After a few weeks of dutifully reading the before-class-time material, I stopped because it felt disjointed and unhelpful during lectures. I would instead look up ideas as I needed them. I would have appreciated more structure and clear expectations with clear deadlines.

What aspects of the course contributed to your intellectual or professional development?
Edgar emphasized making use of external sources. Each topic was accompanied by papers and book references to supplement material covered in class. This encouraged me to look at work outside of the initial delineation of resources.

5. MATH 3061: Modern Geometry, Fall 2018

Modern Geometry is a new course in the Temple catalog, reflecting a change in our geometry offerings for pre-service teachers. Prior to this year the course had been aimed exclusively at students in our secondary education program. To better integrate mathematics education and mathematics majors, as well as meet new licensing needs for middle school teachers, the course was split into a 2000-level Euclidean Geometry course which can serve as an alternative introduction to proof course, and the current version of Modern Geometry, which is aimed at appealing to both secondary-education majors and mathematics majors looking for an elective mathematics course.

I was not involved in the broader re-structuring of the course catalog, but I was involved in developing the new version of MATH 3061 from the moment of preparing a new catalog description onward. I worked with our director of undergraduate studies to select specific content goals and a text. While this course is not formally a "writing intensive" course, I felt that the breadth of geometric topics made available in the catalog description created good opportunities for an extensive research project instead of a written final.

This was my second attempt at incorporating a major writing component (following the course blog in MATH 9072). This project was much more successful and I was able to improve the project during my participation in a faculty learning community about teaching writing intensive courses. In addition to the written component of the project, students were asked to create a (physical or digital) model to illustrate their project. The written papers were excellent, however I found the students struggled somewhat with the model component of the project.

During a spring semester meeting the Temple faculty learning community focused on writing I discussed the difficulties with the model portion of the project, and received very helpful suggestions from a colleague in the College of Fine Arts. In a future course where I use a model as an assessment I will add more structure, parallel to the revision activities used for the paper. Specifically, students will submit a “blueprint” with their outline and bring a model prototype to class for an in-class critique. The colleague who suggested this also offered to meet further about running a critique and observe my class during the activity, should I use a model in a future class at Temple. Unfortunately it is unlikely that I will have that opportunity before leaving Temple, but I plan to seek out similarly generous colleagues in the arts in a future position.

5.1. Syllabus.

Course Overview and Goals. The catalog description of this course is “An introduction to Euclidean and Non-Euclidean geometries with a particular emphasis on theory and proof.” The goals interpret the course as a transformational introduction to these geometries, moving the point of view from measuring things in space to identifying invariant properties of figures under different kinds of symmetry.

Upon Completion of this Course, students will be able to:

- Approach geometric problems from a transformational viewpoint, specifically they will translate between the language of invariants under a transformation and more familiar geometric notions like angle and length.
- Visualize the motion inherent in the transformational approach. Share these visualizations by building models (physical or virtual) and drawing (or etching) pictures.
- Master the use of the group structure of the symmetries of a geometry.
- Use matrix representations to analyze the symmetries of the geometries studied in the class, including: decomposition into reflections, the invariants of a given symmetry, and the
relation between the linear algebra of a matrix (determinant, eigenvalues, etc.) and the
transformation type (reflection, rotation, etc.).
• Appreciate the unifying framework of projective geometry in two dimensions and explain
how the other geometries arise as sub-geometries of the projective plane.
• Use visual and tactile intuition to guide the creation of proofs of geometric facts.

In pursuing these goals students will engage in following mathematical practices:
• Persist, work through perceived failure, and productively self-question.
• Collaborate productively with others and ask good questions.
• Construct examples and counterexamples to investigate new definitions and theorems.
• Read and evaluate existing mathematical proofs, both for correctness and style.
• Create and communicate original proofs.
• Build models that productively build intuition and aid communication.

These goals and practices support the following program learning goals common to all math
majors and minors:
• be open to continuous learning and new ideas
• process and evaluate abstract situations
• communicate mathematical ideas clearly and effectively using written, oral, and digital
  media
• visualize geometric situations
• manipulate abstract objects and ideas

Course Requirements.

Class Participation. Students are expected to attend each meeting of the class and engage with
their peers as well as the material in a collaborative learning environment. Readings will be assigned
before each class session, and reflection questions will be used to guide the day’s activity.

Homework. There will be five homeworks throughout the semester. The reading, reflection, and
in-class work take the place of the “easy” problems you might be used to seeing on an assignment.
The homeworks will be a smaller set of more challenging problems. Solutions will be evaluated both
on their written exposition and on their mathematical content. This is to help develop your writing
skills. Further, after each homework is returned, one problem may be revised and re-submitted
for additional credit. Collaboration on homework problems is encouraged, but you must prepare
your own final write up. It is required that you acknowledge all collaborators. For example, include
something like “Question 2 (with Emma and Earl)”. While collaboration with your classmates is
encouraged, do not consult outside sources (Wikipedia, math.stackexchange). Checking with the
resources in the Annotated Bibliography is OK (as is getting help from the mathematics librarian in
navigating those resources), but if you check with something other than the main textbook include
a citation.

To ensure fairness in grading, I will be using Canvas’ anonymous grading feature. Do not include
your name in the text or file that you upload for your submission. If you collaborated with the whole
class on a problem, acknowledge this collaboration as “(with everyone)” to maintain anonymity.

Reading and Reflection. Before each class meeting students will be assigned readings from the
textbook. Occasionally reading from an alternate source will be supplied via Canvas in PDF format.
Following the reading will be some form of reflection question submitted on Canvas before the class
meeting. Reflections will be graded for completion, but I will also include comments and replies.

Take-home midterm. There will be a take-home midterm given out October 17th and collected
October 19. Collaboration on the take-home midterm is not allowed, and you are not allowed to
consult any sources other than the required textbook (Geometry, Brannan, Esplen and Gray) and
the exam paper.
Geometry Project. The major evaluation in this course will consist of a geometric research project. Students will either research a geometry not covered in this course, or study an invariant not covered in this course. They will prepare a 7 to 13 page survey paper introducing this geometry or invariant to a student who has taken this class but has not seen the particular object involved. To accompany the paper, students will prepare a physical or digital model and give a 20 minute class presentation of their work.

Course Schedule.

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Conics and Euclidean Geometry</td>
</tr>
<tr>
<td>Week 2</td>
<td>Euclidean Geometry and Groups</td>
</tr>
<tr>
<td>Week 3-4</td>
<td>Affine Geometry</td>
</tr>
<tr>
<td>Week 5-6</td>
<td>Projective Geometry</td>
</tr>
<tr>
<td>Week 7</td>
<td>Conics in Projective Geometry</td>
</tr>
<tr>
<td>Week 8</td>
<td>Kleinian Projective Geometry</td>
</tr>
<tr>
<td>Week 9</td>
<td>Spherical Geometry</td>
</tr>
<tr>
<td>Week 10-11</td>
<td>Inversive Geometry</td>
</tr>
<tr>
<td>Week 12, 14</td>
<td>Hyperbolic Geometry</td>
</tr>
<tr>
<td>Week 15</td>
<td>Klein’s View of Geometry &amp; Student Presentations</td>
</tr>
<tr>
<td>Week 16</td>
<td>Student Presentations</td>
</tr>
</tbody>
</table>

Grading of Assignments.

<table>
<thead>
<tr>
<th>Assignments/Activities</th>
<th>% of Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>30%</td>
</tr>
<tr>
<td>Reflections</td>
<td>10%</td>
</tr>
<tr>
<td>Mid-term</td>
<td>15%</td>
</tr>
<tr>
<td>Project Outline</td>
<td>5%</td>
</tr>
<tr>
<td>Project Paper Draft</td>
<td>10%</td>
</tr>
<tr>
<td>Project Talk</td>
<td>10%</td>
</tr>
<tr>
<td>Project Model</td>
<td>5%</td>
</tr>
<tr>
<td>Final Project Evaluation</td>
<td>15%</td>
</tr>
</tbody>
</table>

Letter Grades. Letter grades for the entire course will be assigned as the weighted average of letter grades from individual assignments. This average will be rounded towards the letter grade assigned for the final project evaluation. A passing grade will only be awarded if more than half of the assessments (by weight) have been passed and the final project evaluation receives a passing grade.


Optional Textbooks & Materials.

- Course Annotated Bibliography (on Canvas).
- Matlab (available from computer services).
- Geogebra (available for free on-line).

Course Policies.
Attendance and Tardiness. Your attendance and participation is expected at all iterations of our class meetings and assures that you and your colleagues get the full benefit of taking this class. Your absence and/or coming late or leaving early is a detriment to yourself and to your colleagues. Physical presence alone is not sufficient for attendance, you are present if you arrive on time and participate actively in the day’s classroom activities. Every third unexcused absence (one week of classes) will lower your grade by one letter (e.g. A- to B- for three absences, A- to C- for six).

Academic Honesty/Plagiarism. You are expected to do your own work and any form of academic dishonesty—plagiarism and cheating—is as unacceptable in this course as it is across the University and throughout higher education. Plagiarism is defined in the Bulletin as “the unacknowledged use of another person’s labor, another person’s ideas, another person’s words, and another person’s assistance.” Collaboration will be encouraged in this course. Each assignment and task will specify what kind of collaboration is allowed. In all cases unacknowledged collaboration will be considered cheating, even on assignments that allow unlimited collaboration. If you commit an act of academic dishonesty on an assignment other than the geometry project you will receive an F and be referred to the Office of Student Conduct. If you commit an act of academic dishonesty on any portion of the geometry project you will receive an F in the entire project, and consequently fail the course, as well as be referred to the Office of Student Conduct.

Disability Disclosure Statement. [Standard University Disability Statement]

For visually impaired students: The field of geometry often uses the word “visualize”, however the term refers to the process of building a mental (often 3-dimensional or tactile) spatial model of a situation. In developing specific course materials, I have kept in mind how the geometry can be experienced tacitly, and will be using tactile models in place of visual models wherever possible. Everywhere in the course visualize and tactilize should be treated interchangeably.

5.2. Geometry Project. The students’ work on the geometry project was supplemented by several in class writing workshops focusing on different parts of the writing process. These included a workshop on mathematical sentence clarity and a peer-editing exercise. This section includes the assignment descriptions given to the students, descriptions of the accompanying in-class activities, and the rubrics used to evaluate the project.

Overview. We will not have a final exam. Instead you will undertake a major project where you will investigate a topic related to the class in depth. Your project will involve presenting this topic to other students at a similar level in a variety of ways: a talk, a (digital or physical) model, and a written paper. The topic can take several forms: you could investigate a geometry that we did not investigate in this course (there are lots, and a lot to say about them, you’ll need to be specific), investigate some uncovered aspect of a geometry we did see in this course (for example, trigonometry in hyperbolic or spherical geometry), or investigate particular figures in one or more of the geometries we studied (we said a lot about conics, what about cubics and higher degree equations?). At the end of this page there’s a list of topic ideas, but I encourage you to come up with your own—part of the fun is finding something that speaks to you. One important thing to keep in mind is that your project has a model component. The definition of model will be loosely interpreted, but it must be an object, media work, or finished computer demo that can be interacted with (looking at a playing movie loop counts here) without your intervention.

Collaboration Policy. Everyone is working on a different project topic. You are free to discuss your topic with your classmates, exchange outlines, and proof read for one another. Anything other than writing someone’s paper (or building someone’s model) for them is permitted. If a classmate edits for you, acknowledge them in an "Acknowledgments" section. You can also discuss your project with other majors, once again, acknowledge any editing or other assistance you receive (for example if someone lets you know about an interesting or helpful source for your topic).
Project Components.

Project Topic Note. Write me a short note describing your idea of a project topic. A paragraph or two should do. Mention why you chose it, whether you think it is too narrow or too vague at the moment, if you have any ideas for a model yet (it’s ok if you don’t), and a source you’re thinking of using.

Also mention your availability for a 30 minute conference during the upcoming week (October 29–Nov 2).

Project Topic Conference. Based on the availability you include with your note, we will schedule a one-on-one conference in my office. In this conference I’ll help you refine your project topic idea (broadening it if you think it’s too narrow, or narrowing it if you think it’s too vague), recommend some additional sources, and help you refine your sense of where the project is going. Bring the source you mentioned.

Project Outline. Write a “Mathematical Outline” of your planned paper (this might also function as an outline for your talk). This outline should include the sections you plan to use, with section titles and a bullet point or two about the purpose of the section, followed by complete, precise statements of definitions, lemmas, and theorems that you plan to include. If a theorem has many alternate proofs, maybe include a bullet point after the theorem about how you’ll prove it. Indicate which statements are drawn from which sources by citations (or, if you are synthesizing a source to develop a section, indicate that in the section bullet point) and include a list of references (this can grow as your project continues if you need to).

In class sentence clarity workshop. Bring a mostly-finished rough draft to the class two days before the rough draft is due. I will present elements of both English and mathematical statement clarity. Then, with a partner, read through your draft, locate unclear sentences and workshop revisions.

Project Draft. Submit a typed draft of your paper. The paper is a 7-13 page (around 5-9 pages without diagrams, but you have lots of space for pictures) article introducing a student who has taken our class to your topic.

Your draft does not need to be completely polished, but I expect that all of the planned content (as presented in your outline, possibly modified for space reasons) should be there. If you plan to use pictures of your model but your model is not yet complete, just leave a space for it and a descriptive caption. Other figures and diagrams should be present. At this stage, since this is a draft, your figures and diagrams can be drafts too (scans of sketches).

I’ll evaluate the draft on my writing rubric, but the grade will be based on content and direction, not polish. I’ll also give you extensive feedback that we will use in the last week of classes in a revision activity.

In class revision activity one: “Writing into the gaps”. In my feedback on your papers I will identify specific gaps. These will be gaps in exposition (sudden transitions, unintroduced material, etc.) or gaps in the mathematical work (uncited claims, shortcuts in proofs, etc.). Bring your draft with my feedback. In class, work on filling the gaps I’ve identified, and work with a partner on proposed revisions to fill the gaps.

In class revision activity two: “Polish”. This activity will occur during our last class meeting before presentations begin. Bring an updated draft based on the revisions from the previous activity. This should be close to your final version. I will introduce strategies for line editing (checking grammar, punctuation, and mathematical notation). Work through your paper using these strategies, and consult with a partner (or me) should you have a question or need to choose among several possible revisions.
Project Model. Turn in your model at the start of class on December 7th. If you’re talking on December 10th I’ll bring your model to class then.

The model should illustrate a concept from your project. It can be digital or physical, and should provide for some kind of interaction. “Looking at” counts as an interaction, but I expect a higher quality finished product compared to a model that is interactive (compare the cardboard of the pin and thread to the 3d printed Dandelin spheres). If you would like, I will arrange for your model to be on display in the department.

Project Talk. Give an in-class talk (mini-lesson) on the topic of your paper. You will probably not be able to present everything in your papers in twenty minutes, so you’ll need to make choices about what to present in detail, what to mention, and what to leave out entirely. The talk should give the listener a feel for the ideas involved in your topic, and entice them to want to read your paper.

If you would like to give a slide talk, please e-mail me your slides before your presentation so I can have them set-up on the computer. If there are other special things you’d like (big paper, string, etc.) let me know and I’ll see what can be arranged (within reason).

Final Paper. Submit the final draft of your paper introducing your topic to a student who has taken our class but not seen your topic. The paper should be 7-13 pages, you can go over the page limit if your text is 5-9 pages (not including works cited) and you have a lot of pictures. Sources used should be cited in a works cited section; use any citation style you like as long as you use it consistently throughout the paper.

I expect this submission to be highly polished, incorporating the feedback from the rough draft and edited to a high level of clarity using the strategies practiced in class.

Guidelines for Figures
- Figures (photographs, graphs, and diagrams) should be formatted centered, with the text not word wrapped around it.
- Figures should be accompanied by a caption and numbered, like “Figure 1: A dog”. If a figure has multiple images broken into sub figures they should be sublettered, like “Figure 1(a): A cute dog”, “Figure 1(b): A cuter dog”.
- If you did not create the figure yourself the caption should include a citation indicating where the figure is from and the source should be included in your works cited.
- If your figure is hand-drawn the scan should be in high-contrast with the background suppressed, so that the background appears as solid white. This might require tracing your diagram in ink.

Guidelines for Equations and Notation
- Use an equation editor (either the one in canvas and save the image, or LaTeX, or MS Word’s) to get access to standard notation, and format all mathematical equations appropriately.
- If an equation is on its own line it should be centered. If it is part of a derivation the derivation should have the equality and inequality signs aligned.

Rubrics. I will evaluate the components of the project on the following rubrics. Each criteria will be rated with one of the following ratings: Mastery, Proficiency, Needs Improvement, Not Assessable; accompanied by my written feedback. The rubric criteria follow.

Writing Rubric.
- Statement Clarity
  Clearly states the theorem to be proved, quantity to be calculated, property to be verified, or problem to be solved. Includes all assumptions in the statement.
- Audience
  The writing is aimed at the appropriate audience.
• Diagram Labels
  All diagrams, tables, and other visual representations are clearly labeled and connected
to the exposition.
• Variable Introduction
  All mathematical items (variables, functions, etc.) that appear in the discussion are
named and introduced.
• Written Mechanics
  Spelling, grammar and punctuation are correct.
• Mathematical Written Mechanics
  Symbols are only used as part of formulas, never as replacements for English words.
  Terminology is used correctly.
• Proofiness
  Proves (or cites a proof of) every major claim made. Derives (or cites the derivation of)
every formula used in a calculation.
• Mathematical Correctness
  The proofs, constructions, and calculations presented are mathematically correct.
• Mathematical Creativity
  The explorations, generated conjectures, and proof techniques are creative and novel.

Model Rubric.
• Craft quality
  The model is well made as a piece of craft work. This doesn’t mean expensive materials:
it is the difference between a precise origami cube and a crumpled up wad of paper crushed
into a cube shape. If the model is software, the software should be polished (no random
crashes, no “don’t click it that way”) and the source code commented.
• Conceptual clarity
  The model speaks for itself (at least at an intuitive level) about the ideas involved, just
by interacting with it, without requiring explanation.
• Mathematical accuracy
  The model is as true as possible to the mathematics represented, it is not a cartoon or
sketch.
• Mathematical relevance
  The model illustrates a concept or concepts directly related to the project’s topic.

Talk Rubric.
• Organization
  The order of presentation was logical, and had a good flow from topic to topic.
• Board/Slides used effectively
  Board: writing was clear and legible, flowed from left to right. Slides: slides were not
overcrowded with information, slides were not rushed or skipped.
• Time
  The talk did not go over-time.
• Motivation
  The reason for considering the topic and its relationship to the course material was
presented.
• Main Ideas
  The main ideas of the proof(s) presented were highlighted and correct.
• Background
  An appropriate amount of background was presented, given the time and audience.
• Interest
The presenter captured audience interest with stage presence and interactivity.

- Questions
  The presenter attended to audience questions without getting derailed.
- Communication
  The presenter spoke clearly and at an audible volume.

5.3. **Student evaluations.** The Temple electronic student feedback forms are only used in courses with an enrollment of five or more. Instead, for this course, the director of undergraduate studies conducted an informal non-anonymous student evaluation of teaching. All four students submitted replies.

*Student 1:*

1. **What aspects of the course or the instructor’s approach contributed most to your learning?**
   The readings were sometimes very hard to understand but during class most concepts were made very clear. The various models and visuals were very helpful. I liked the format of the midterm. The main focus on just a few types of Geometry and their similarities is helpful.

2. **What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?**
   I think that for a first time taking geometry in college the content was quite difficult and writing geometric proofs is still something I’d have trouble doing on my own.
   The homework was also quite difficult because it often did not relate to the textbook or class resources.
   These are both problems that are not that serious and geometry has never been my strong suit.

3. **Please comment on the instructor’s sensitivity to the diversity of the students in the class.**
   Edgar was very kind and relatable as a professor. He treated us as his peers and was understanding and considerate of our busy schedules.

*Student 2:*

1. **What aspects of the course or the instructor’s approach contributed most to your learning?**
   The visual aids helped a lot and I don’t think I would have done well in this class if the size was any bigger. I really liked the smallness of the class because it made the learning more like a conversation so I could ask a lot of clarifying questions.
   Also I felt (not sure if this is what really happened) that you graded our work with our effort in mind. There were certain assignments that I hadn’t felt like I produced high quality answers on but I did work for hours and hours on it. I appreciate that my hard work was recognized (even if I wasn’t always “right”).

2. **What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?**
   I really didn’t like the reading reflections. Honestly the text was so hard to comprehend that I barely understood each reading. I tried to answer the reflections to the best of my understanding, but I was honestly defeated by the textbook early on. The lectures did clarify a bit, but there was a gap that I felt. I attribute my confusion to my complete lack of geometry knowledge prior to the start of the course.

3. **Please comment on the instructor’s sensitivity to the diversity of the students in the class.**
   Has always been sensitive and I appreciated that you took the time to address Brett Kavanaugh being admitted onto the Supreme Court. None of my other professors took the time to do this.
Student 3:

(1) What aspects of the course or the instructor’s approach contributed most to your learning?

He gave more feedback than any professor I’ve ever had. It was always constructive and if we didn’t understand what he meant he was always willing to meet in person. I appreciated that he let us revise the homeworks and do so many rounds of editing our papers in class because it gave me the opportunity to use his feedback and improve.

(2) What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?

I wish his reflection assignments would have covered a little more so I could make sure I got everything I was supposed to out of the reading.

I also think when designing this course he had too high of expectations for us. That being said, he did a great job revising his lesson plans and he was always willing to slow down if we were confused.

(3) Please comment on the instructor’s sensitivity to the diversity of the students in the class.

I really liked how he often talked to us about how his lessons could be revised for visually impaired students. This was helpful as a future teacher because geometry is such a visual subject.

Student 4:

(1) What aspects of the course or the instructor’s approach contributed most to your learning?

Fortunately we had a small class, so even if all of us had questions Dr. Bering was able to slow the class down to embellish on certain topics to help us understand better. The class was more of a workshop style instead of lecture style which meant our learning was active and more hands on.

(2) What aspects of the course or the instructor’s approach would you change to improve the learning that takes place in the course?

Linear algebra needs to be taught with more visual aids or with a less calculation based view. I really needed a lot of my skills from linear algebra but because I wasn’t clearly given the geometric reasoning behind the calculations I never saw the importance behind it.

(3) Please comment on the instructor’s sensitivity to the diversity of the students in the class.

Dr. Bering has always made every effort to accommodate us and to be fair with deadlines and grading.

6. MATH 1022: Precalculus, Spring 2019

MATH 1022 is a coordinated, multi-section course. As a course instructor I was responsible for classroom instruction and setting weekly summative assessments in the form of quizzes and homework. The course content and major assessments were set by the department coordinator.

The course coordinator for MATH 1022 is an advocate for active learning in our department. With another Temple colleague, she had previously received a local grant to re-design MATH 1022 to run as a flipped class, and makes these materials available to all MATH 1022 instructors. I made use of her materials instead of preparing my own in delivering this course. We consulted several times about the results of various lessons, and I contributed revisions at the end of the semester.

I achieved the content goals of the course and I am proud of the work my students did. Of the Precalculus sections offered in the Spring 2019 semester, mine had the second highest class average, the highest proportion of students performing well enough to continue to Calculus (74.29% compared to the coursewide 56.73%), and lowest DFW rate (22.86% compared to the coursewide 37.31%).

The transition to a flipped classroom can cause friction with students who are not expecting it. Based on my student feedback form, my students that semester did not internalize the learning successes indicated by the course assessments. Recent literature on running flipped classrooms has
found this to be a common experience, and suggests that devoting more class time to discussing the reasons behind my pedagogic choices will aid students in recognizing their own strengths.

6.1. Selected feedback quotes.

“There was a lot of group work and things to keep our focus while learning. It wasn't like any other math course I've had before.”

“I really liked the preformatted packets that were given for each lesson. It was eye pleasing to study when I needed to look back at my notes.”

“The design of the class left a lot of room for me to ask questions on topics that I was not clear on. It also left room for me to help my classmates with material I felt strongly on and they did not. I found this very helpful because I think teaching something is an effective way to master material.”

7. MATH 4063: Topology I, Spring 2019

MATH 4063 is a senior elective for majors, including majors in the secondary education program. I was asked to teach this course one week before the semester started, and found myself in a panic. Preparing to teach an active course from scratch requires significant preparation. Thankfully, my colleague Matthew Stover is a proponent of Inquiry Based Learning, and offered some assistance. He first put me in touch with Richard Canary, Ralf Spatzier, and Sarah Koch, who shared the University of Michigan IBL materials for a similar course. Then, throughout the semester, Stover visited my classroom to check-in with how I was adapting to the method. In an IBL course, course material is developed and presented by the students based on guiding structured activities provided by the instructor. The students discover the proofs of the major theorems, in lieu of using a textbook. This was my first time teaching an IBL course as I had been intimidated by the prospect of preparing suitable guiding materials; I was grateful for his mentorship. With materials at hand I was able to focus on delivery: building identity and creating community.

The course was a great success, the students grew into the material and exceeded my expectations mathematically. Moreover, throughout the course they each refined personal styles and learned to find collaborators with complementary strengths. The class had a strong community, and I would regularly find most of my students in the undergraduate lounge discussing the material, refining ideas and debating not only the correctness of alternative proofs, but matters of taste in exposition and approach.

Being able to use the University of Michigan materials as a starting point was a great help, but they did require some adaptation to the specific setting at Temple. One major change that I made, based on input from Stover, was the separation of exposition from the in-class worksheets. Stover had observed that his students would often immediately jump to the first exercise, and then circle back to the exposition piecemeal. I hoped to encourage active reading by my students, so I incorporated the reading and reflection system I previously used in Modern Geometry. In revising the exposition to fit this model I took the opportunity to include direct addresses to the reader pointing out when they should check something in the margin, and some good questions to ponder as they read. The goal of these revisions was to have the readings develop active reading habits in the students.

Content-wise, the largest departure I made from the Michigan materials was the inclusion of infinite products. This was due to the strength of my particular class, and was not a strict curricular requirement. However, I used the opportunity to develop IBL content of my own, in consultation with Stover. I’ve included the module as a sample in this section.

7.1. Syllabus.
**Course Overview and Goals.** Topology is a fundamental area of mathematics that provides a foundation for mathematical analysis. Once a set $X$ has a topology we have enough structure on the space to know what it means for a functions to be continuous on $X$. Once we have continuous functions we can build a large part of the familiar theory from real analysis. The goal of this course is to introduce you to the world of topology, where emphasis is placed on careful reasoning, and on understanding and constructing proofs.

Upon completion of this course, students will be able to:

- Abstract key properties of the real numbers to the general settings of metric and topological spaces.
- Construct examples and counterexamples to illustrate theorems and definitions—particularly the sometimes counter-intuitive flexibility of abstract definitions.
- Specialize abstract definitions to specific settings and prove alternative characterizations.
- Carry out abstract topological arguments guided by a mature intuition for connectedness, closeness, and continuity.
- Continue the study of mathematical analysis or topology in an independent study, undergraduate research experience, or graduate school.

In pursuing these goals students will engage in following mathematical practices:

- Persist, work through perceived failure, and productively self-question.
- Collaborate productively with others and ask good questions.
- Read and evaluate existing mathematical proofs, both for correctness and style.
- Create and communicate original proofs.

These goals and practices support the following program learning goals common to all math majors and minors:

- be open to continuous learning and new ideas
- process and evaluate abstract situations
- communicate mathematical ideas clearly and effectively using written, oral, and digital media
- manipulate abstract objects and ideas

**Course Requirements.**

**Class Participation.** Students are expected to attend each meeting of the class and engage with their peers as well as the material in a collaborative learning environment. Readings will be assigned before each class session, and reflection questions will be used to guide the day’s activity.

This course is taught in an inquiry based style, active participation in class activities makes up the bulk of instruction and is essential. However, I ask that you do not work on the in-class worksheets out of class, as that defeats much of the purpose of IBL.

**Homework.** There will be a list of problems updated regularly through the semester. Some problems will be optional and some required, and I will collect these periodically. The exact due dates depend on our pace through the material. The homeworks will be an opportunity to build your library of examples, explore related tangents, and develop as a mathematical writer. Solutions will be evaluated both on their written exposition and on their mathematical content. This is to provide you with guidance and opportunity to develop your writing skills. Collaboration on homework problems is encouraged, but you must prepare your own final write up. It is required that you acknowledge all collaborators. For example, include something like “Question 2 (with Emma and Earl)”. While collaboration with your classmates is encouraged, do not consult outside sources.

To ensure fairness in grading, I will be using Canvas’ anonymous grading feature. Do not include your name in the text or file that you upload for your submission. If you collaborated with the whole class on a problem, acknowledge this collaboration as “(with everyone)” to maintain anonymity.
Reading and Reflection. Before some class meetings, at the start of new worksheets, students will be assigned short readings via Canvas. Following the reading will be some form of reflection question submitted on Canvas before the class meeting. Reflections will be graded for completion.

Take-home midterm. There will be a take-home midterm given out March 12th and collected March 14th. Collaboration on the take-home midterm is not allowed, nor are you allowed to consult any sources other than the readings and your own completed worksheets and notes.

Grading of Assignments. The grade for this course will be determined according to the following formula:

<table>
<thead>
<tr>
<th>Assignments/Activities</th>
<th>% of Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation</td>
<td>40%</td>
</tr>
<tr>
<td>Mid-term</td>
<td>20%</td>
</tr>
<tr>
<td>Final</td>
<td>20%</td>
</tr>
<tr>
<td>Homework</td>
<td>20%</td>
</tr>
</tbody>
</table>

Course Schedule. The course will be self paced, adjusting to our rate of collective discovery and progress. We will follow this general outline of topics, and a * indicates which modules will conclude with homework collection.

(1) Metric Spaces
   (a) Definitions and examples
   (b) Openness, Closedness, Closures, and Boundaries
   (c) Continuity *
   (d) Sequences, Compactness, and the metric Bolzano-Weierstrass theorem
   (e) Product Spaces *

(2) Topological Spaces
   (a) Definitions and examples
   (b) Openness, Closedness, and Continuity
   (c) Product Spaces *
   (d) Connectivity
   (e) Compactness *
   (f) Products of compact sets
   (g) Separation Axioms

(3) Extended topics (time permitting)
   We will decide on these as a class if we’ve covered the above schedule.
7.2. Sample reading, reflection, and worksheet.

Math 4063  
PRODUCT SPACES  
SPRING 2019

Throughout this reading, we will be considering a collection \((X_\alpha, \mathcal{T}_\alpha)\) of topological spaces indexed by \(\alpha \in A\).

In our consideration of metric products, we proved that for a finite product \(X \times Y\), a subset \(W \subseteq X \times Y\) is open if and only if for every \((x, y) \in W\) there is an open set \(U_{(x,y)} \subseteq X\) containing \(x\) and an open set \(V_{(x,y)} \subseteq Y\) containing \(y\) such that \(U_{(x,y)} \times V_{(x,y)} \subseteq W\). This suggests one way to put a topology on the product \(\prod_{\alpha \in A} X_\alpha\).

**Definition.** The **box topology** on the product \(\prod_{\alpha \in A} X_\alpha\) is the topology \(\mathcal{T}_b\) with basis

\[
\mathcal{B}_b = \left\{ \prod_{\alpha \in A} U_\alpha | U_\alpha \in \mathcal{T}_\alpha \right\}
\]

Check in the margin that this collection satisfies the necessary conditions to be a basis.

We also proved that for a product of metric spaces \(X \times Y\) the projection maps \(\pi_X\) and \(\pi_Y\) are continuous. Using this as a starting point, we can define a topology on \(\prod_{\alpha \in A} X_\alpha\) that has enough open sets so that each projection map is continuous, but no extras. This will use the **subbasis** construction.

**Definition.** The **product topology** on the product \(\prod_{\alpha \in A} X_\alpha\) is the topology \(\mathcal{T}\) with subbasis

\[
\mathcal{S} = \bigcup_{\alpha \in A} \{\pi_\alpha^{-1}(U) | U_\alpha \in \mathcal{T}_\alpha\}
\]

By construction, with this topology every projection map is continuous. The basis \(\mathcal{B}\) generated from \(\mathcal{S}\) is the collection of all finite intersections of sets from \(\mathcal{S}\). For a fixed index \(\alpha\), and open sets \(U_\alpha, V_\alpha \subset X_\alpha\), we have \(\pi_\alpha^{-1}(U_\alpha) \cap \pi_\alpha^{-1}(V_\alpha) = \pi_\alpha^{-1}(U_\alpha \cap V_\alpha)\) which is again an element of \(\mathcal{S}\).

Therefore an element of \(\mathcal{B}\) can be described as follows. Let \(J \subseteq A\) be a finite subset of indicies, so that \(J = \{\alpha_1, \ldots, \alpha_n\}\). Then

\[
B = \pi_{\alpha_1}^{-1}(U_{\alpha_1}) \cap \cdots \cap \pi_{\alpha_n}^{-1}(U_{\alpha_n})
\]

is an element of \(\mathcal{B}\). Since a point \((x_\alpha) \in \prod X_\alpha\) is in \(B\) when \(x_{\alpha_i} \in U_\alpha\) for the coordinates \(\alpha_i \in J\) and not otherwise restricted, we can describe \(B\) as the following product

\[
B = \prod_{\alpha \in J} U_{\alpha}
\]

Where \(U_\alpha = X_\alpha\) if \(\alpha \notin J\). The point of this is that we get an equivalent definition of the product topology in terms of a basis.

**Definition.** The **product topology** on the product \(\prod_{\alpha \in A} X_\alpha\) is the topology \(\mathcal{T}\) with basis

\[
\mathcal{B} = \left\{ \prod_{\alpha \in A} U_\alpha | U_\alpha \in \mathcal{T}_\alpha \text{ and } U_\alpha = X_\alpha \text{ for all but finitely many } \alpha \right\}
\]

This equivalent description of the product topology makes it clear that for a finite product, the box topology and the product topology are equal. It will turn out, however, that they are different for infinite products and that the box topology is badly behaved. When working with infinite products, unless otherwise specified, we will use the product topology.

**Reflection Question.** Above I alluded to the “bad behavior” of the box topology. Last week, we pondered the question about a subset \(A \subseteq X\) of a topological space.

Suppose \(x \in \text{cl}(A)\), does there exist a sequence in \(A\) converging to \(x\)?
The box topology gives us new spaces to test this question on. Let \( X = \prod_{n=1}^{\infty} [0, 1] \) and \( A = \{(x_i) \in X | \forall i : x_i > 0\} \). Consider the point \( x = (0, 0, 0, \ldots) \). Is \( x \) in the closure of \( A \) in the box topology on \( X \)? Is there a sequence in \( A \) converging to \( x \) in the box topology on \( X \)? (Hint: consider the open neighborhood \( U = \prod_{n=1}^{\infty} [0, x_i^{(i)})\)).

Once you’re satisfied with your answers try to think up a property of products of metric spaces that you suspect the space \( X \) will not have with the box topology. Try to provide a proof, if you can; if not, some intuition behind your choice. Follow the reflection guidelines as you draft your reply.

**Math 4063 Products Spring 2019**

**Theorem.** Suppose that \((X_\alpha, T_\alpha)\) is a collection of topological spaces, \((Z, T_Z)\) is a topological space, and \(f_\alpha: Z \to X_\alpha\) is a collection of functions. The map \(f: Z \to \prod_\alpha X_\alpha\) given by \(f(z) = (f_\alpha(z))\) is continuous when \(\prod_\alpha X_\alpha\) is given the product topology if and only if each \(f_\alpha: Z \to X_\alpha\) is continuous.

**Proof.** Exercise.

The next example illustrates that this theorem is false for the box topology on the product.

**Example.** Let \( X = \prod_{i=1}^{\infty} \mathbb{R} \) be a countable product of copies of the real numbers, each copy given the metric topology. Consider the diagonal function \( \Delta: \mathbb{R} \to X \) defined by \( \Delta(t) = (t, t, t, \ldots) \). By the previous theorem, when \( X \) is given the product topology \( \Delta(t) \) is continuous. Prove that \( \Delta(t) \) is not continuous when \( X \) is considered with the box topology.

Part of our motivation for pursuing topological spaces was the recognition that the different metrics we put on a finite product gave rise to equivalent topologies. It is therefore important to check that the product topology and the metric topology coincide.

**Lemma.** Suppose that \((X, d_X)\) and \((Y, d_Y)\) are metric spaces. Prove that \(T_{d_2}\) for the metric \(d_2\) on \(X \times Y\) agrees with the product topology on \(X \times Y\) coming from the topological spaces \((X, T_{d_X})\) and \((Y, T_{d_Y})\).

**Proof.** Exercise.

Using the subbasis definition of the product topology, for a product \(X \times Y\) the projection maps are continuous by definition. Continuity is not the only property of functions between topological spaces we’ve investigated.

**Question.** Given a product \(X \times Y\) of topological spaces, are the projection maps open? Are they closed?

Metric spaces are Hausdorff, so we might wonder if products preserve Hausdorffness.

**Lemma.** Suppose \((X, T_X)\) and \((Y, T_Y)\) are Hausdorff. Then \(X \times Y\) with the product topology is Hausdorff.

**Proof.** Exercise.

**Question.** Is the converse true? Prove or give a counterexample.
7.3. Selected feedback quotes.

“Edgar’s pedagogy is one that I feel should be normalized in mathematics it would be to all of our benefit. Out of every single math course I’ve taken here at Temple, Edgar’s Topology I class has been the only one to expose me to what collaboration looks like in our field. There is great value in learning how to work with others, and I sincerely believe that our class came to appreciate this fact, as well as the many things we’ve learned from each other. When people were reluctant to share their rough ideas before coming up with a proof, Edgar employed various tactics to get us all comfortable with talking things through (some of these tactics left us with no choice but to do so). Edgar also has a knack for taking our ideas and hinting towards how we can turn them into refined proofs, which is indicative of his command over the subject. It’s also refreshing to have a professor that values all ideas, rather than imposing preferred proof strategies onto us. He is likewise one of the only professors I’ve witnessed who consistently tries to help us improve our writing, and is always ready to give thoughtful and constructive feedback.”

“The course setup allowed students to think critically about the material instead of simply blind copying notes from the board. It allowed students to creatively explore their own depths of proofiness and develop proofs that wouldn’t necessarily have been thought of before. Homework and midterm assessments and assignments were met with extreme rigor and expected students to collaborate with one another, but also allowed students to choose their own problems under some guidelines based on their specific backgrounds in mathematics. For example, some students in the course did not have a strong background in group theory or even ring theory from modern algebra, so although those problems were included on assignments, they were only there as one of many options. In terms of the actual instructor, as said before, I have not met a professor who was more passionate about an area of mathematics before. His approach to student learning clearly shows that he has an interest in, and strong ability toward promoting active student learning in his teaching style in and out of the classroom. My view of Edgar was almost as a mentor toward pushing my to do my very best work in mathematical writing, both officially on graded assignments and in office hours and casual conversation outside the classroom. When shortcuts or gaps were taken, it was noted and met with questions that would necessarily help students recognize and fill in those shortcuts or gaps.”

8. Undergraduate Research Supervision

Research experience can provide an undergraduate with an unequalled opportunity to develop a mathematical identity, provided the student is prepared to take advantage of the experience. In designing my upper division courses I have endeavored to provide that preparation. The reading and reflection activities are aimed to develop the active reading skills needed for effective research. In MATH 4063: Topology I, the classroom was centered around collaborative problem solving, conjecture forming, and exposition. The geometry project of MATH 3061 developed all aspects of the research process.

While at Temple I supervised one undergraduate research student, Olivia Bayer. The supervision was as a part of the Francis Velay Fellowship program at Temple, and the Velay program included career mentoring and social activities for the fellows. Olivia found these beneficial to her participation, and in future undergraduate supervision I will include similar activities if they are not part of an existing program.

In addition to building a community of undergraduate research, in my experience it is vital for an undergraduate researcher to be invited to the scholarly life of the department. The Temple
Geometry & Topology group runs a summer reading seminar for its graduate students each summer, and I had Olivia participate in this seminar. As a result, in addition to the formal mentoring and career development activities, Olivia built robust professional relationships with the Geometry & Topology graduate students. Through these friendships she continues to receive valuable informal mentoring that enhances and reinforces the formal program. Based on this positive impact, I will incorporate any future undergraduate research program as a part of the larger research group.