Extra problems for Homework 12
Due Wednesday, 4/11/07

1. Let $M$ and $N$ be closed, connected, oriented manifolds of dimension $n$. Let $M\#N$ be their connected sum. Prove that

$$\tilde{H}_k(M\#N) \cong \tilde{H}_k(M) \oplus \tilde{H}_k(N),$$

for all $0 \leq k < n$. *Hint: be careful when $k = n - 1$!*

2. Let $M_g$ and $M_h$ be the oriented surfaces of genus $g$ and $h$, respectively.

(a) When $g \geq h$, construct a degree–1 map $f : M_g \to M_h$.

(b) Prove that when $g > 0$, every map from $S^2$ to $M_g$ has degree 0. *Hint: first, do the case when $g = 1$.

*Extra credit.* Show that when $g < h$, every map from $M_g$ to $M_h$ has degree 0.