1. There is a standard way to glue together two (connected) manifolds $M$ and $N$ of the same dimension. Remove an open ball $B^n$ from each of $M$ and $N$, and glue $M\setminus B^n$ to $N\setminus B^n$ along the two $(n - 1)$ dimensional boundary spheres. The resulting manifold is called the connected sum of $M$ and $N$, and is denoted $M\#N$.

   a) Prove that when $n \geq 3$, $\pi_1(M\setminus B^n) \cong \pi_1(M)$.
   
   b) Prove that when $n \geq 3$, $\pi_1(M\#N) \cong \pi_1(M) \ast \pi_1(N)$.

2. Let $M$ be a manifold of dimension $n \geq 4$, and let $C$ be a knot (i.e., an embedded circle) in $M$.

   a) Prove that $\pi_1(M\setminus C) \cong \pi_1(M)$.
   
   b) Is the hypothesis that $n \geq 4$ necessary?

3. Let $X = S^2 \cup A$, where $A$ is an axis connecting the north and south poles of $S^2$. Find a cell complex structure on $X$, and use it to compute the fundamental group.

4. Do problem 8 on page 53 of Hatcher.

5. Do problem 14 on page 54 of Hatcher.

*General hint:* for some of these problems, van Kampen’s theorem will be more useful than Proposition 1.26 (the application to cell complexes).