1. Let \( S \) be a compact surface in \( \mathbb{R}^3 \), and let \( p \in S \) be the point of \( S \) that is furthest from the origin. Prove that the Gaussian curvature at \( p \) is positive.

2. Let \( S \) be a compact surface in \( \mathbb{R}^3 \), and suppose that its Euler characteristic is \( \chi(S) \leq 0 \). Prove that there must be points on \( S \) where the Gaussian curvature is positive, zero, and negative.

3. Let \( \gamma(t) \) be a simple closed curve in the \( x - z \) plane, whose \( x \)-coordinate is always positive. Let \( S \) be the surface of revolution obtained by rotating \( \gamma \) about the \( z \)-axis.
   
   (a) Compute the Euler characteristic of \( S \).
   
   (b) What standard surface is \( S \) homeomorphic to?

4. Let \( S \) be a surface homeomorphic to a sphere, such that the Gaussian curvature of \( S \) is everywhere positive. Let \( \gamma_1 \) and \( \gamma_2 \) be simple closed geodesics on \( S \), such that the interior of \( \gamma_1 \) is region \( R_1 \) and the interior of \( \gamma_2 \) is region \( R_2 \). Prove that regions \( R_1 \) and \( R_2 \) must intersect. 
   
   \textit{Hint:} if they do not intersect, think about what’s left of \( S \) after removing both \( R_1 \) and \( R_2 \).