1. Let $S$ be the hyperboloid $z = 2xy$.

(a) Prove that at the origin, $S$ has mean curvature $H = 0$ and Gaussian curvature $K = -4$.

(b) Let $T$ be a surface obtained by turning $S$ about the origin until the principal vectors at $(0,0,0)$ become tangent to the $x$-axis and $y$-axis. Describe a parametrization of $T$.

2. Let $S$ the ellipsoid described by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(a) Let $R \subset S$ be the region where $x \geq 0, y \geq 0, z \geq 0$. Sketch the image of $R$ under the Gauss map. What is its area?

(b) Compute $\iint_R K \, dA = \iint_R K |\sigma_u \times \sigma_v| \, du \, dv$, without doing any calculus. *Hint:* examine the proof of Theorem 7.1 in the book.

(c) Prove that the integral of Gaussian curvature over the entire surface is $\iint_S K \, dA = 4\pi$.

3. Let $\gamma(t) = (x(t), z(t))$ be a unit-speed curve, such that $x(t) > 0$ for all $t$. Let $S$ be the surface obtained by revolving $\gamma$ about the $z$-axis.

(a) Prove that $\gamma(t) = (x(t), 0, z(t))$ is a geodesic in $S$.

(b) Let $t_0$ be a number such that $z(t)$ reaches a local maximum at $t_0$, and let $\delta(s)$ be the circle obtained by revolving the point $(x(t_0), z(t_0))$ about the $z$-axis. Is $\delta(s)$ a geodesic in $S$?

*Hint:* this should only need the definition of a geodesic. Drawing pictures will help!

4. The Little Prince lives on a planet whose radius is 5m, and tends to a rose garden whose area is 100$m^2$. If the rose garden is an equilateral triangle on this planet, what is the angle at each corner?