Math 4061 Homework 1
Due Thursday, 1/28/10

1. For $t \in \mathbb{R}$, let $\delta(t)$ be a parametrized curve which does not pass through the origin. Let $t_0$ be a time at which $\delta$ comes closest to the origin. Prove that $\delta(t_0) \cdot \delta'(t_0) = 0$.

2. Let $\gamma$ be a parametrized curve such that $\gamma''(t) = 0$ for all $t$. What can you say about the shape of $\gamma$?

3. Consider the graph of the absolute value function $y = |x - 1|$ in $\mathbb{R}^2$.
   a) Find a parametrization of this curve.
   b) Find a four-times-differentiable parametrization of this curve.
   c) Does there exist a regular parametrization?

4. Consider a 3–dimensional logarithmic spiral, parametrized by
   $$\alpha(t) = (e^{-t} \cos t, e^{-t} \sin t, e^{-t}).$$
   a) Compute the arclength between $\alpha(0)$ and $\alpha(r)$.
   b) Prove that the total length of $\alpha$, as $t$ ranges in $[0, \infty)$, is finite.

5. Consider the parametric curve $\gamma : [1, \infty) \to \mathbb{R}^2$, parametrized by
   $$\gamma(t) = \left(\frac{\sin t}{t^2}, \frac{1}{t}\right).$$
   a) Does $\gamma$ have a unit-speed reparametrization?
   b) Prove that the total length of $\gamma$ is finite. Hint: Can you estimate the integral instead of computing it?