MATH 2043 ADDITIONAL HOMEWORK PROBLEMS

A12: VECTORS AND THE GEOMETRY OF SPACE

1. Let $\overrightarrow{AB} = \langle 2, 3, 1 \rangle$. If the coordinates of A are $(2, 1, 5)$, what are the coordinates of the point B?

2. Let $\mathbf{a} \cdot \mathbf{b} = 3$ and $\mathbf{a} \cdot \mathbf{c} = 7$. Find $\mathbf{a} \cdot (3\mathbf{b} - \mathbf{c})$.

3. Let $\mathbf{a} \cdot \mathbf{b} = 3$ and $|\mathbf{a}| = 4$, $|\mathbf{b}| = 2$. Find $|\mathbf{a} - 2\mathbf{b}|$.

4. Let $\mathbf{a} = \langle x, y, 1 \rangle$, $\mathbf{b} = \langle 1, -1, 3 \rangle$. Assume that $|\mathbf{a}| = \sqrt{14}/2$ and $\mathbf{a} \cdot \mathbf{b} = 5$. Find all possible values of x and y.

5. Let $\mathbf{a} \times \mathbf{b} = \langle 2, -1, 5 \rangle$ and $\mathbf{a} \times \mathbf{c} = \langle 1, 4, 2 \rangle$.

   (a) Find $\mathbf{a} \times (\mathbf{a} + \mathbf{b} - \mathbf{c})$.

   (b) Find $\mathbf{a} \cdot \langle 2, -1, 5 \rangle$ and $\mathbf{a} \cdot \langle 1, 4, 2 \rangle$.

   (c) Find a unit vector parallel to $\mathbf{a}$.

6. Let $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 3 \rangle$ and let $|\mathbf{a}| = \sqrt{7}$, $|\mathbf{b}| = 2\sqrt{2}$. Find all possible values of the angle θ between $\mathbf{a}$ and $\mathbf{b}$.

7. Let Q be the foot of the perpendicular from the point $P(1, 0, -3)$ to the plane $x + 2y + 3z = 2$ (*i.e.*, the point Q lies in the plane and the line through P and Q is perpendicular to the plane). Find the coordinates of the point Q.

A13: VECTOR FUNCTIONS

1. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t\sqrt{t-1}\,\mathbf{i} + t\sin(\pi t)\,\mathbf{j} + \ln t\,\mathbf{k}$ and $\mathbf{r}(2) = \mathbf{i} + 3\mathbf{k}$

2. Let C be the curve $\mathbf{r}(t) = \langle t^2, \sin t, \cos t \rangle$, $0 \le t \le 1$. Find $\int_C f(t)ds$ where $f(t) = \dfrac{t}{4t^2 + 1}$.

A14: PARTIAL DERIVATIVES

1. Let $f(x, y) = e^{x^2 + 2y - 3}$.

   (a) Find f_x and f_y.

   (b) Find the linearization $L(x, y)$ of f at the point $(-1, 1)$.

   (c) Use your answer to part (b) to approximate $f(-1.04, 0.95)$.

2. Let $f(x, y, z) = x\sqrt{y + 4z}$.

   (a) Find f_x, f_y, and f_z.

   (b) Find the linearization $L(x, y, z)$ of f at the point $(3, 1, 2)$.

   (c) Use your answer to part (b) to approximate $f(3.02, 0.9, 2.1)$.

A15: MULTIPLE INTEGRALS

1. Calculate the double integral over a given rectangle R.

   (a) $\displaystyle\iint_R x^2 \cos y e^{x \sin y}\, dA, \quad R = \{(x, y)|1 \le x \le 2, 0 \le y \le \pi/2\}.$

   (b) $\displaystyle\iint_R \frac{x+y}{1+y^2}\, dA, \quad R = \{(x, y)|0 \le x \le 2, 0 \le y \le 1\}.$

2. Use cylinderical coordinates to find the volume of the solid that lies in the first octant above the cone $z = \sqrt{x^2 + y^2}$ and below the paraboloid $z = 2 - x^2 - y^2$.

3. Convert to spherical coordinates and evaluate: $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\, dzdydx.$

4. Evaluate $\displaystyle\iiint_D e^{-(x^2+y^2+z^2)^{3/2}}\, dV$, where D is the region that lies below the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$.

A16: VECTOR CALCULUS

1. Find the mass of a thin wire bent into the shape of the curve $\mathbf{r}(t) = \langle \sin(2t), \sin t, \cos t\rangle,\ 0 \le t \le \pi/4$, if the linear density is $\rho(x, y, z) = x(z^2 - y^2)$.

2. $\mathbf{F}(x, y) = (xy^2 + 1)\mathbf{i} + (x^2y - 2y)\mathbf{j}, \quad C : \mathbf{r}(t) = \langle t + \sin(\frac{1}{2}\pi t), t + \cos(\frac{1}{2}\pi t)\rangle, \quad 0 \le t \le 1.$

   (a) Verify that $\mathbf{F}$ is a conservative vector field.

   (b) Find a function f such that $\mathbf{F} = \nabla f$

   (c) Use part (**a**) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

3. $\mathbf{F}(x, y, z) = y^2z\mathbf{i} + \left(2xyz - \frac{z}{y^2}\right)\mathbf{j} + \left(xy^2 + \frac{1}{y} - 2z\right)\mathbf{k}. \quad C : \mathbf{r}(t) = \langle\sqrt{t}, t + 1, t^2\rangle, \quad 0 \le t \le 1$

   (a) Verify that $\mathbf{F}$ is a conservative vector field.

   (b) Find a function f such that $\mathbf{F} = \nabla f$

   (c) Use part (**a**) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

4. $\mathbf{F}(x, y, z) = (yze^{xyz} + 3x^2z)\mathbf{i} + (xze^{xyz} + z\cos(yz) + 2)\mathbf{j} + (xye^{xyz} + y\cos(yz) + x^3 - 2z)\mathbf{k}$, $C : \mathbf{r}(t) = \langle t + 1, t^2, t^3 + 2\rangle, \quad 0 \le t \le 1.$

   (a) Verify that $\mathbf{F}$ is a conservative vector field.

   (b) Find a function f such that $\mathbf{F} = \nabla f$

   (c) Use part (**a**) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.