2.1 Solving Linear Equations:

- **Equation:** is a statement that one algebraic expression is equal to another.
- **Linear equation:** is an equation that can be written in the form $ax+b=0$.
- **Solve:** means find all the values of the unknown x for which the statement is true.

- **Equation-Solving Strategies:**

When solving an equation, you must isolate the variable on one side of the equation using one or more of the following steps.

1) Simplify an expression by removing parentheses. Then combine **like terms** **(**combine numbers or expressions with the same variable names).
2) Add or subtract the same number or expression to (from) both sides of the equation.
3) Multiply or divide both sides of the equation by the same nonzero number.
4) Check your answer. This is the final step and the most often skipped step, yet it is probably the most important step in the process. With this step you will be able to know whether or not you got the correct answer. We verify the answer by plugging the result from the previous steps into the **original** equation. It is very important to plug into the original equation since you may have made a mistake in the very first step which leads you to an incorrect answer.

Ex 1: Solve the following equation.

(a) $3(x+2)-2=4x$

Solution

To solve the given equation we proceed as follows:

$3(x+2)-2=4x$	Given equation
$3x+6-2=4x$	Remove parentheses
$3x+6-2-4x=4x-4x$	Subtract $4x$ from both sides
$-x+4=0$	Combine like terms
$-x=-4$	Isolate term containing x
$x=4$	Divide both sides by -1

Therefore, $x=4$ is the solution to the given equation. Check the solution by substituting $x=4$ in the original equation:

$$3(4+2)-2=4(4)\Rightarrow 16=16$$

(b) $5x+2(x-1)=7(x-1)$

- **Solving an equation involving fractions or fractional expressions:**
  To solve, you need to clear the denominators by multiplying **BOTH** sides of the equation by the least common denominator (LCD). Then follow the steps above.

Ex 2: Solve the equation.

$$\frac{x+5}{2}+\frac{2x-1}{5}=5$$

Solution

To solve the given equation we proceed as follows:

Clear the denominators by multiplying **BOTH** sides of the equation by the least common denominator (LCD), which is 10.

$$10\left(\frac{x+5}{2}+\frac{2x-1}{5}\right)=10\cdot 5 \quad \text{Multiply both sides by LCD}$$

$$5(x+5)+2(2x-1)=50 \quad \text{Simplify each term}$$

$$5x+25+4x-2=50 \quad \text{Remove parentheses}$$

$$9x+23=50 \quad \text{Combine like terms}$$

$$9x=27 \quad \text{Subtract 23 from both sides}$$

$$x=3 \quad \text{Divide both sides by 9 to solve for } x$$

You can check the answer in the original equation.

NOTE: If the equation contains any fractions use the least common denominator to clear the fractions. We will do this by multiplying both sides of the equation by the LCD. Also, remember that if there are variables in the denominators of the given fractions, you have to identify values of the variable which will give a zero denominator. We will need to avoid these values in our solution, because division by zero is NOT permitted.

Ex 3: Solve the equation.

$$\frac{2x}{x+3}=\frac{3}{x-10}+2$$

Solution

In this case it looks like the LCD is $(x+3)(x-10)$ and it also looks like we will need to avoid $x=-3$ and $x=10$ to make sure that we do not get division by zero.

$$(x+3)(x-10)\left(\frac{2x}{x+3}\right)=\left(\frac{3}{x-10}+2\right)(x+3)(x-10)$$

$$2x(x-10)=3(x+3)+2(x+3)(x-10)$$

$$2x^2-20x=3x+9+2\left(x^2-7x-30\right)$$

At this point let us pause and acknowledge that we have an x^2 in the work here. Do not get worried. Sometimes these will show up **temporarily** in these problems. You should only worry about it if it is still there after we finish simplifying the problem.

$$2x^2 - 20x = 3x + 9 + 2x^2 - 14x - 60$$
$$-20x = -11x - 51$$
$$51 = 9x$$
$$\frac{51}{9} = x$$

Notice that the x^2 did in fact cancel out. Now, if we did our work correctly $x = \frac{17}{3}$ should be the solution since it is not either of the two values that will give division by zero.

Ex 4: Solve the equation.

(a) $\frac{2}{x+1} = 4 - \frac{2x}{x+1}$

Solution

The LCD for this equation is $\boldsymbol{x+1}$ and we will need to avoid $\boldsymbol{x=-1}$ so we don't get a zero in the denominator. Here is the work for this equation.

$$\left(\frac{2}{x+1}\right)(x+1) = \left(4 - \frac{2x}{x+1}\right)(x+1)$$
$$2 = 4(x+1) - 2x$$
$$2 = 4x + 4 - 2x$$
$$2 = 2x + 4$$
$$-2 = 2x$$
$$-1 = x$$

So, we once again arrive at the single value of x that we needed to avoid so we didn't get division by zero. Therefore, this equation has **no solution**.

(b) $\frac{2x}{x-1} + \frac{x+1}{x-2} = \frac{3x-1}{x-1}$

(c) $\frac{x}{x-1} = \frac{1}{x-1} + 1$

*** Solving linear equations containing Radical expressions:**

The main idea is to **isolate** the term containing the radical and then **square** both sides of the equation to eliminate the radical.

If, after squaring, there is still another radical, repeat the process. Then solve the resulting equation.

EX 5: Solve the equation.

$$\sqrt{x} = 3$$

Solution

Here the term containing the radical is already isolated. All we need is to square both sides and solve the resulting equation.

$$\left(\sqrt{x}\right)^2 = (3)^2$$
$$x = 9$$

Again, you need to check answers.

EX 6: Solve the equation.

a) $\sqrt{3x+1} - 2 = 0$

Solution

$\sqrt{3x+1} - 2 = 0$	Original equation
$\sqrt{3x+1} - 2 + 2 = 0 + 2$	Isolate the radical term by adding 2 to both sides
$(\sqrt{3x+1})^2 = (2)^2$	Square both sides
$3x + 1 = 4$	Linear equation; solve
$3x + 1 - 1 = 4 - 1$	Subtract 1
$3x = 3$	Divide by 3
$x = 1$	

b) $\sqrt{4x+11} - \sqrt{1-x} = 0$

c) $\sqrt{3x+2} - \sqrt{3x-1} = 1$

- **Solving linear equations with absolute values:**

  To solve $|P| = a$**, solve** $P = a$ **or** $P = -a$ (Absolute value property)

  If a is 0, then $P = 0$.

  If a is negative, the equation $|P| = a$ has no solution.

EX 7: Solve the equation.

$$|2x - 5| = 9$$

Solution

There really isn't much to do here other than using the absolute value property. All we need to note is that in the property above, *p* represents whatever is on the inside of the absolute value bars and so in this case we have,

$$2x - 5 = -9 \quad \text{or} \quad 2x - 5 = 9$$

At this point we have two linear equations that are easy to solve.

$$2x = -4 \quad \text{or} \quad 2x = 14$$
$$x = -2 \quad \text{or} \quad x = 7$$

So, we have two solutions to the equation $x = -2$ and $x = 7$.

EX 8: Solve each of the following.

(a) $|10x - 3| = 0$

(b) $|5x + 9| = -3$

Solution

(a) Let us approach this one from a geometric standpoint. This is saying that the quantity in the absolute value bars has a distance of zero from the origin. There is only one number that has the property and that is zero itself. So, we must have,

$$10x - 3 = 0$$
$$x = \frac{3}{10}$$

In this case we get a single solution.

(b) Now, in this case let us recall that we noted at the start of this topic that If a is negative, the equation $|P| = a$ has no solution. In other words, we can't get a negative value out of the absolute value. That means there is **no solution** to this equation.

****DO NOT FORGET TO ALWAYS CHECK YOUR WORK!!**

In 1 – 7, solve the equations for *x*.

1. $3x + 11 - (6x - 11) = 0$

2. $5(x - 2) + 3(3x - 1) = 4(x - 3) + 7x$

3. $\frac{2}{x^2 - 9} - \frac{3}{x - 3} = \frac{1}{x + 3}$

4. $\frac{3x + 5}{x^2 + 3x + 2} = \frac{1}{x + 2} + \frac{2}{x + 1}$

5. $\sqrt{3x - 2} = 5$

6. $\sqrt{4x + 8} - \sqrt{4x + 3} = 1$

7. $|4x + 3| = 8$

**** Please Do HW problems as well.**

2.2 Solving Linear Inequalities:

Inequality problems involve the following special symbols and notation, read from left to right.

$\leq$ means "**less than or equal to**"

$\geq$ means "**greater than or equal to**"

$<$ means "**strictly less than**"

$>$ means "**strictly greater than**"

Solutions to inequality problems can be written in three different ways.

For example, the statement "all numbers less than or equal to 5" can be written using

1) "**inequality form**" $x \leq 5$

2) a "**number line graph**"

Note: 1. The **included endpoint** $x = 5$ is indicated with a "**full circle**".

2. An "**empty circle**" is used when an **endpoint is not included.**

3) "**interval form**" $(-\infty, 5]$

Notes: 1. The **included endpoint** $x = 5$ is indicated with a "**bracket**".

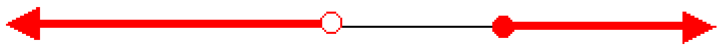
2. **Parentheses** are used when an **endpoint is not included**.

3. The symbol $-\infty$ represents "**negative infinity**."

Another example is the statement "numbers which are less than - 1 or greater than or equal to 2."

* "**inequality form**" $x < -1$ or $x \geq 2$

* a "**number line graph**"



* "**interval form**" $(-\infty, -1)$ or $[2, \infty)$

***Basic Solving Rules for Inequalities:**

When solving inequality problems, we will be able to use many of the same rules we have already used when solving equations.

Remember, **our goal is to perform a sequence of operations so we can isolate the desired variable term**.

Example 1: Solve the following

a) $3x - 7 > x + 5$

Solution: $3x > x + 12$ Add 7 to both sides

$2x > 12$ Subtract x from both sides

$x > 6$ Divide both sides by 2

Interval notation: $(6, +\infty)$

b) $x - \frac{1}{2} \leq -\frac{x}{2} + 2$

Solution: $2x - 1 \leq -x + 4$ Multiply both sides by 2

$2x \leq -x + 5$ Add 1 to both sides

$3x \leq 5$ Add x to both sides

$x \le \frac{5}{3}$ Divide both sides by 3

Interval notation: $\left(-\infty, \frac{5}{3}\right]$

c) $5x + 4 \ge 5(x+1)$

Solution: $5x + 4 \ge 5x + 5$ Apply distributive property on right

$5x \ge 5x + 1$ Subtract 4 from both sides

$0 \ge 1$ Subtract 5*x* from both sides

Notice here, since 0 is not greater than or equal to 1, we must conclude that there is **no solution. The solution is an empty set ϕ.**

d) $x + 2(x-1) < 3x + 5$

Solution: $x + 2x - 2 < 3x + 5$ Apply distributive property on left

$3x - 2 < 3x + 5$ Combine like terms on left

$3x < 3x + 7$ Add 2 to both sides

$0 < 7$ Subtract 3 from both sides

Notice here, since 0 is less than 7, we must conclude that the solution consists of the set of **All real numbers.**

Interval notation: $(-\infty, +\infty)$

In order to isolate the variable in an inequality, we must remember **two new solving rules.**

Rule 1: When you **multiply or divide by a negative number**,
you must **reverse the direction of the inequality**.

For example:

solving $-3x > 9$

leads to $\frac{-3x}{-3} < \frac{9}{-3}$

so we get $x < -3$ as our solution.

Example 2: Solve $5x - 3(2x+1) \geq 0$

Solution: $5x - 6x - 3 \geq 0$ Apply distributive property on left

$-x - 3 \geq 0$ Combine like terms on left

$-x \geq 3$ Add 3 to both sides

$x \leq -3$ Divide both sides by *-1*, and reverse the direction of the inequality

Example 3: Solve $-3x + 7 \leq 2(4x+3)$

Rule 2: Unlike solving equations, **clearing out a variable denominator in an inequality problem introduces possible confusion over whether to reverse the direction of the inequality.**

Therefore, **we will never multiply or divide by a variable term**, because it will be too hard to keep track of reversals in the inequality.

Instead **we will construct a special number line which will help us keep track of all possible outcomes in the problem.**

* Solving Inequalities with Rational expressions:

Example 4: Solve the following.

a) $\dfrac{x+1}{x-5} \leq 0$

Solution:

1) Get a zero on one side and write the other side as a single rational inequality. This has already been done for us here.

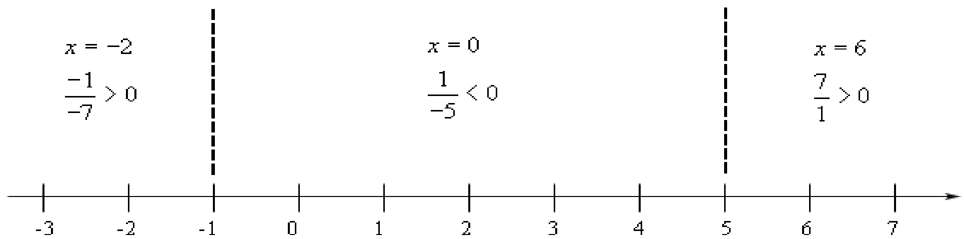
2) Factor the numerator and denominator as much as possible. Again, this has already been done for us in this case.

3) Determine where both the numerator and the denominator are zero. In this case these values are.

Numerator: $x = -1$ **Denominator:** $x = 5$

Now, we need these numbers for a couple of reasons. First, just like with polynomial inequalities these are the only numbers where the rational expression *may* change sign. So, we'll build a number line using these points to define ranges out of which to pick test points just like we did with polynomial inequalities.

There is another reason for needing the value of *x* that makes the denominator zero. Here we have a rational expression and that means we need to **avoid division by zero** and so knowing where the denominator is zero will give us the values of *x* that we need to avoid. Here is the number line for this inequality.



We need regions that make the rational expression negative. That means the middle region. Also, since we have an "or equal to" part in the inequality we also need to include where the inequality is zero, so this means we include $x=-1$. Notice that we will also need to avoid $x=5$ since that gives division by zero. (Remember division by zero is NOT ALLOWED).

The solution for this inequality in interval Notation is: $[-1,5)$.

b) $\dfrac{2x+1}{x-2}>0$

c) $\dfrac{x+8}{x+3}>2$

* Linear Inequalities with Absolute Values:

In the previous section we solved equations that contained absolute values. In this section we want to look at inequalities that contain absolute values. We will need to examine two separate cases.

*** Inequalities Involving < and ≤ :**

As we did with equations let's start off by looking at a fairly simple case.

$$|p|\le 4$$

This says that no matter what *p* is it must have a distance of no more than 4 from the origin. This means that *p* must be somewhere in the range,

$$-4\le p\le 4$$

We could so a similar inequality with the < and get a similar result.

In general we have the following formulas to use here,

If $|P| < a$ **, then** $-a < P < a$

If $|P| \le a$ **, then** $-a \le P \le a$

Example 5*:* Solve each of the following.

(a) $|2x-4| < 10$

(b) $|9x+2| \le 1$

(c) $|3-2x| \le 5$

Solution:

(a) $|2x-4| < 10$

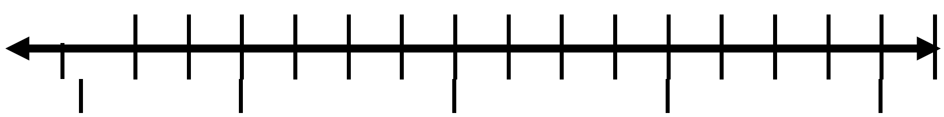
To solve for *x*, we need to use the formula. As with equations *p* simply represents whatever is inside the absolute value bars. So, with this first one we have,

$$-10 < 2x - 4 < 10$$

Now, this is nothing more than a simple double inequality that we need to solve

$$-6 < 2x < 14$$
$$-3 < x < 7$$

Now, try graphing the solution.

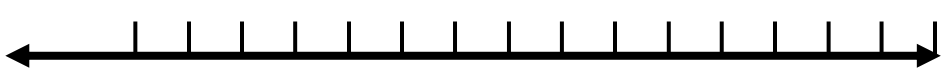


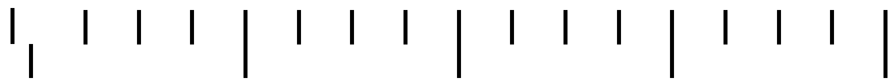
The interval notation for this solution is $(-3,7)$.

(b) $|9x+2| \le 1$

$$-1 \le 9x + 2 \le 1$$
$$-3 \le 9x \le -1$$
$$-\frac{1}{3} \le x \le -\frac{1}{9}$$

Now, try graphing the solution.





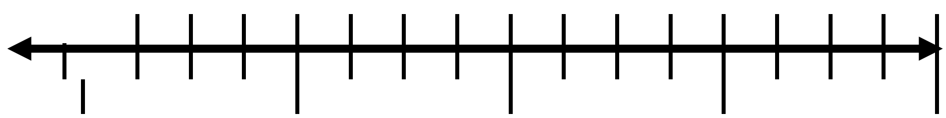
The interval notation is $\left[-\frac{1}{3},-\frac{1}{9}\right]$

(c) $|3-2x|\le 5$

We need to be a little careful with solving the double inequality with this one, but other than that it is pretty much identical to the previous two parts.

$$-5\le 3-2x\le 5$$
$$-8\le -2x\le 2$$
$$4\ge x\ge -1$$

In the final step do not forget to switch the direction of the inequalities since we divided everything by a negative number.
Now, try graphing the solution.



The interval notation for this solution is $[-1,4]$.

* **Inequalities Involving > and $\ge$:**
Once again let's start off with a simple number example.

$$|p|\ge 4$$

This says that whatever p is it must be at least a distance of 4 from the origin and so p must be in one of the following two ranges,

$$p\le -4 \qquad \text{or} \qquad p\ge 4$$

Before giving the general solution we need to address a common mistake that students make with these types of problems. Many students try to combine these into a single double inequality as follows,

$$-4\ge p\ge 4$$

While this may seem to make sense we can not stress enough that THIS IS NOT CORRECT!! Recall what a double inequality says. In a double inequality we require that both of the inequalities be satisfied simultaneously. The double inequality above would then mean that p is a number that is simultaneously smaller than -4 and larger than 4. This just doesn't make sense. There is no number that satisfies this.

These solutions must be written as two inequalities.

In general we have the following formulas to use here,

If $|P|>a$ **, then** $P<-a$ **or** $P>a$

If $|P| \geq a$ **, then** $P \leq -a$ **or** $P \geq a$

Example 6: Solve each of the following.

(a) $|2x-3| > 7$

(b) $|6x+10| \geq 3$

(c) $|2-6x| > 10$

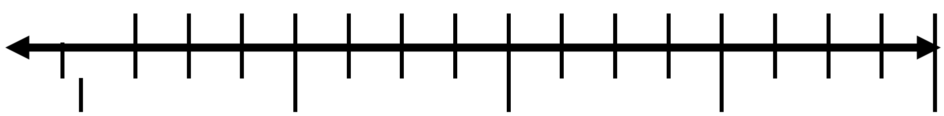
Solution:

(a) $|2x-3| > 7$

Again, *p* represents the quantity inside the absolute value bars so all we need to do here is plug into the formula and then solve the two linear inequalities.

$2x-3 < -7$	or	$2x-3 > 7$
$2x < -4$	or	$2x > 10$
$x < -2$	or	$x > 5$

Graph the solution on the number line.



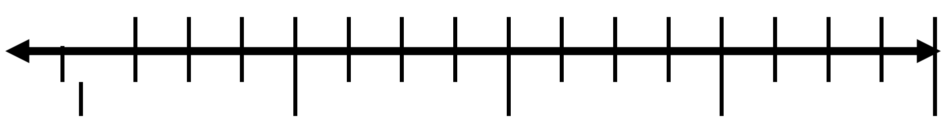
The interval notation : $(-\infty,-2)$ or $(5,\infty)$.

(b) $|6x+10| \geq 3$

Let's just plug into the formulas and go here,

$6x+10 \leq -3$	or	$6x+10 \geq 3$
$6x \leq -13$	or	$6x \geq -7$
$x \leq -\frac{13}{6}$	or	$x \geq -\frac{7}{6}$

Graph the solution on the number line.



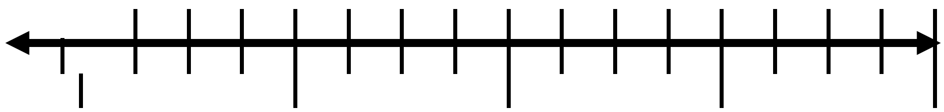
The interval notation : $\left(-\infty,-\frac{13}{6}\right]$ or $\left(-\frac{7}{6},\infty\right]$.

(c) $|2-6x|>10$

$2-6x<-10$	or	$2-6x>10$
$-6x<-12$	or	$-6x>8$
$x>2$	or	$x<-\frac{4}{3}$

Notice that we had to switch the direction of the inequalities when we divided by the negative number!

Graph the solution on the number line.



The interval notation for these solutions is $\left(-\infty,-\frac{4}{3}\right)$ or $(2,\infty)$.

SUMMARY:
*** Interval notation:**

Inequality	Graph	Interval Notation
$a\le x\le b$		$[a,b]$
$a<x<b$		(a,b)
$a\le x<b$		$[a,b)$
$a<x\le b$		$(a,b]$
$x>a$		(a,∞)
$x\ge a$		$[a,\infty)$
$x<b$		$(-\infty,b)$

$x \le b$  $(-\infty, b]$

****Remember that a bracket, "[" or "]", means that we include the endpoint while a parenthesis, "(" or ")", means we don't include the endpoint.**

****Also remember to reverse the direction of the inequality when we multiply or divide both sides of the inequality by a negative.**

In 1 – 5, solve the inequalities and graph the solutions. Express the solutions in interval notation.

1. $-4x - 5 \le 0$

2. $\dfrac{x+2}{x-3} \le 0$

3. $\dfrac{x-1}{2x+3} > 0$

4. $|3x+4| < 3$

5. $|5x-4| \ge 2$

6. $-|3x-2| \le 6$

7. $|-2-x| \ge 9$

8. $\dfrac{x-2}{x+3} > -4$

9. $\dfrac{x+2}{x+4} \le 3$

**** Please Do HW problems as well.**

2.3 Applications of linear equations:

Our main goal in this section is to construct an equation which describes the relationships among the relevant quantities in a given problem.

In this section your success in solving a problem depends on your ability to construct an appropriate linear equation.

Polya (a mathematician, known as the master of problem-solving) created his famous **four-step process for problem solving,** which is used all over to aid people in problem solving:

Step 1: Understand the problem.

Sometimes the problem lies in **understanding the problem**. If you are unclear as to what needs to be solved, then you are probably going to get the wrong results. In order to show an understanding of the problem, you, of course, **need to read the problem carefully**. Sounds simple enough, but some people try to start solving the problem before they have read the whole problem. Once the problem is read, you **need to list all the components and data that are involved**. This is where you will be assigning your variable.

Step 2: Devise a plan (translate).

When you **devise a plan (translate)**, you come up with a way to solve the problem. Setting up an equation, drawing a diagram, and making a chart are all ways that you can go about solving your problem.

Step 3: Carry out the plan (solve).

The next step, **carry out the plan (solve)**. This is where you solve the equation you came up with in your 'devise a plan' step.

Step 4: Look back (check and interpret).

In problem solving it is good to **look back (check and interpret) your answer(s).** Basically, check to see if you used all your information and that **the answer makes sense.** If your answer does check out, make sure that you write your final answer with the correct labeling.

*** DISTANCE Problems:**

The standard formula we will be using here is:

$d = rt$

Where $d =$ distance traveled by an object along a given path, in miles.

$r =$ average rate of speed, in miles per hour (mph).

$t =$ time of travel, in hours (hr).

- You may use the following table to organize the given information.

	Distance (mi)	Rate (mph)	Time (hr)

WARNING: When solving a word problem, you can add distances and you can add times, but you cannot add rates. **<u>Think about it:</u> if you drive 20 mph on one street and 40 mph on another, does that mean you averaged 60 mph??**

Example 1:
It takes you 4.5 hours to drive from your home to your favorite weekend get away, which is 315 miles away. What is your average speed?

Solution:
Let's apply the **four-step process for problem solving**

Step 1: Understand the problem.

Make sure that you read the question carefully several times. Since we are looking for speed, we can use the distance/rate formula:

$$d = rt$$

The variables in this formula represent the following:

d = distance
r = rate
t = time

Step 2: Devise a plan (translate).

In this problem,

$d = 315$
r = ? = this is the variable we are looking for
$t = 4.5$
Plugging the values into the formula $d = rt$ we get:

$$315 = r(4.5)$$

Step 3: Carry out the plan (solve).

$$315 = r(4.5)$$

$$\frac{315}{4.5} = \frac{r(4.5)}{4.5}$$

$$70 = r$$

Step 4: Look back (check and interpret).

If you go at a rate of 70 miles per hour for 4.5 hours, you would travel 315 miles.

FINAL ANSWER:

The average speed is 70 mph.

Example 2: An airplane flying with the wind can cover a certain distance in 2 hours. The return trip against the wind takes 2.5 hours. How fast is the plane and what is the speed of the wind, if the one-way distance is 600 miles.

Step 1: Understand the problem.

Make sure that you read the question carefully several times.

Since we are looking for two different rates (speeds), we will let

x = rate/speed of the plane

y = the rate/speed of the wind

Since this is a rate/distance problem, it might be good to organize our information using the distance formula.

Keep in mind that the wind speed is affecting the overall speed.

When the plane is with the wind, it will be going faster. That rate will be *x + y*.

When the plane is going against the wind, it will be going slower.

That rate will be *x - y*.

Hint: You may use the following Chart to organize the given information:

	Distance (mi)	Rate (mph)	Time (hr)
With the wind	600	$x + y$	2
Against the wind	600	$x - y$	2.5

Step 2: Devise a plan (translate).

Since we have two unknowns, we need to build a system with two equations.

Equation (1):

Rate times Time is equal to Distance

$$(x+y)(2) \quad = \quad 600$$

Equation (2):

Rate times Time is equal to Distance

$$(x-y)(2.5) \quad = \quad 600$$

Now, $\text{Rate} = \dfrac{distance}{time}$

$(x+y) = \dfrac{600}{2}$ **and** $(x-y) = \dfrac{600}{2.5}$

$(x+y) = 300$ $\qquad$ $(x-y) = 240$

Step 3: Carry out the plan (solve).

Add, $x+y+x-y=300+240$ **to cancel** y

$$2x=540$$

Solving for x **we get:**

$$\frac{2x}{2}=\frac{540}{2}$$

$$x=270$$

Solve for second variable.

Using equation (1) to plug in 270 for x **and solving for** y **we get:**

$$x+y=300$$

$$270+y=300$$

$$270+y-270=300-270$$

$$y=30$$

Step 4: Look back (check and interpret).

You will find that if you plug the values x =270 and y = 30 into either equation, this is a solution to BOTH of them.

Final Answer:
The airplane speed is 270 mph and the air speed is 30 mph

*** PRICING Problems:**

*** MARKUP**

S= C + rC

Where S= selling price

C= cost (original price)

r = mark up rate %

***EXAMPLE:**

1) A calculator has been marked up 15% and is being sold for $78.50. How much did the store pay the manufacturer of the calculator?

2) The manager of an electronics store buys an MP3 player for $200 and sells the player for $250. Find the mark up rate.

3) The sale price for a digital phone is $49. This price is 25% of the original price. Find the original price.

*** MARKDOWN**

$$S = C - rC$$

Where S= selling price

C= cost (original price)

r = mark down rate % (discount rate)

EXAMPLES:

1) A clock radio that regularly sells for $24.95 is on sale at a 20% discount. What is the sale price?

2) A shirt is on sale for $15 and has been marked down 35%. How much was the shirt being sold for before the sale?

*** WORK Problems:**

In solving a work problem, the goal is to determine the **time** it takes to complete a task.

If a painter can paint a room in 4 hrs, then in 1 hr the painter can paint $\frac{1}{4}$ of the room.

If two painters are working together:

(Amount of work done by the 1^{st}) +(Amount of work done by the 2^{nd}) = (Amount of work done by both)

$$\left(\frac{1}{t_1}\right) + \left(\frac{1}{t_2}\right) = \left(\frac{1}{t_b}\right)$$

Where $t_1 =$ time it takes the first, working alone, to finish the work.

$t_2 =$ time it takes the second, working alone, to finish the work.

$t_b =$ time it takes **both working together** to finish the work.

EXAMPLES:

1) A park has two sprinklers that are used to fill a fountain. One sprinkler can fill the fountain in 3hrs, whereas a second sprinkler can fill the fountain in 6 hrs. How long will it take to fill the fountain with both sprinklers operating?

2) Two oil pipes can fill a small tank in 30 min. Using one of the pipelines would require 45 min to fill the tank. How long would it take the second pipeline, working alone, to fill the tank?

*** MIXTURE Problems:**

This type of problems involves mixing solutions of different percentages to get a new percentage. The solution will consist of a secondary liquid mixed in with water. The secondary liquid can be alcohol or acid for instance.

The formula we will be using here is:

$$A = MC$$

Where $A =$ Amount of mixture, in liters/milliliters/ounces etc….

$M =$ Medium, Number of liters/ milliliters/ounces/gallons,

$C =$ Concentrate/ Strength in %

- You may use the following table to organize the given information.

	Part 1	Part 2	Mixture
Concentrate %			
Medium (liters)			
Amount of concentrate			

For example: If 3 liters of 30% alcohol are mixed with 2 liters of 50% alcohol, what will be the concentration of the mixture?

Example 3: How many gallons of 20% alcohol solution and 50% alcohol solution must be mixed to get 9 gallons of 30% alcohol solution?

Step 1: Understand the problem.

Make sure that you read the question carefully several times.

Since we are looking for two different amounts, we will let

***x* = the number of gallons of 20% alcohol solution**

***9-x* = the number of gallons of 50% alcohol solution**

Step 2: Devise a plan (translate).

Let's use the following table to organize the given data.

	Part 1	Part 2	Mixture
Concentrate %	20%	50%	30%
Medium (gallons)	x	$(9-x)$	9
Amount of concentrate	$.2x$	$.5(9-x)$	$.3(9)$

Step 3: Carry out the plan (solve).

From the last row of the table, we add the Amount of concentrate in each part to obtain

$$.2x + .5(9-x) = .3(9)$$
$$.2x + 4.5 - .5x = 2.7$$
$$-.3x = 2.7 - 4.5$$
$$-.3x = -1.8$$
$$x = 6$$

We need to find the number of gallons needed of the 50% solution, by plugging x=6 in $9 - x = 9 - 6 = 3$

Step 4: Look back (check and interpret).

You will find that if you plug the values x =6 in the equation, it will work.

$$.2x + .5(9-x) = .3(9)$$

Final Answer:
6 gallons of 20% solution and 3 gallons of 50% solution

Practice problems:

1) John drove 150 miles in $3\frac{1}{2}$ hours. His average speed for the first 90 miles was 45 mph. What was his average speed for the last 60 miles?

2) An airplane traveling against a steady wind takes 2 hrs to make a 500-mile trip. The return trip in the direction of the wind takes only 1 hr and 40 min. What is the speed of the plane in still air, and what is the speed of the wind?

3) With the aid of the current, Joe can row a canoe 3 miles in 12 minutes. Against the current, he requires 18 minutes to row the same distance. How fast does Joe row in still water, and how fast is the current?

4) A boat traveled from a harbor to an island at an average speed of 20 mph. The average speed on the return trip was 15 mph. The total trip took 3.5 hours. How long did it take to travel to the island?

5) Two liters of 40% alcohol are mixed with 4 liters of another alcohol to form a 50% alcohol solution. What is the strength of the added alcohol?

6) A pharmacist needs 70 liters of a 50% alcohol solution. She has available a 30% alcohol solution and an 80% alcohol solution. How many liters of each solution should she mix to obtain 70 liters of a 50% alcohol solution?

7) A chemist wishes to make 2 liters of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

8) Forty ounces of a 30% gold alloy are mixed with 60oz of a 20% gold alloy. Find the percent concentration of the resulting gold alloy.

2.4 Midpoint and Distance formulas:

** The Midpoint Formula:*

Sometimes you need to find the point lies exactly between two other points. This middle point is called the "midpoint".

If you are given two numbers, you can find the number that is exactly between them by averaging them, by **adding them together and dividing by two**.

The Midpoint Formula is: Given the two points (x_1, y_1) and (x_2, y_2), the midpoint between these points is given by the formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex 1: Find the midpoint between (–1, 2) and (3, –6).

Apply the Midpoint Formula:

$$\left(\frac{-1+3}{2}, \frac{2+3}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) = (1, -2)$$

So the answer is ***Midpoint*=*M* = (1, –2)**.

Ex 2: Find the value of x_1 so that (–2, 2.5) is the midpoint between (x_1, 2) and (–1, 3).
Apply the Midpoint Formula:

$$\left(\frac{x_1 + (-1)}{2}, \frac{2+3}{2}\right) = (-2, 2.5)$$
$$\left(\frac{x_1 - 1}{2}, \frac{5}{2}\right) = (-2, 2.5)$$
$$\left(\frac{x_1 - 1}{2}, 2.5\right) = (-2, 2.5)$$

We need to figure out what x_1 is, in order to make the *x*-values work; so equate the x-values:

$$\frac{x_1 - 1}{2} = -2$$
$$x_1 - 1 = -4$$
$$x_1 = -3$$

So the answer is x_1 **= –3.**

*** The Distance Formula:**

Given the two points (x_1, y_1) and (x_2, y_2), the distance between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The subscripts indicate that there is a "first" point and a "second" point; that is, you have two points. Whichever one you call "first" or "second" is up to you.

Ex 3: **Find the distance between (–1, 2) and (3, –6).**

Ex 4: Find the radius of a circle, given that the center is at (2, –3) and the point (–1, –2) lies on the circle.

NOTE: Here the problem does not explicitly state that you need to use the Distance Formula; instead, you have to notice that you need to find the distance, and then remember the Formula.

The radius is the distance between the center and any point on the circle, so find the distance:

$$d = \sqrt{(2-(-1))^2 + (-3-(-2))^2} = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{(3)^2 + (-1)^2}$$
$$= \sqrt{9+1} = \sqrt{10}$$

Ex 5: Show that the points $P_1(2,3)$, $P_2(8,3)$, and $P_3\left(5,3+3\sqrt{3}\right)$ are the vertices of an equilateral triangle.

Solution: Here you have to find the distance between P_1 **and** P_2, P_1 **and** P_3, and P_2 **and** P_3, and check whether the sides are of the same length or not.

Ex 6: Given $P_1(2,-3)$, $P_2(4,1)$ and $P(x,5)$, find the value of x for which $d(P,P_1) = d(P,P_2)$.

Now, try the following problems:

1) A diameter of a circle has endpoints $P_1(-1,4)$ and $P_2(10,1)$. Is the point $P(8,7)$ on the circle?

2) Find the distance between the point $(5,0)$ and the midpoint of the line segment from $(1,2)$ to $(7,-8)$.

3) Show that the points $P_1(2,1)$, $P_2(4,-1)$, and $P_3(7,4)$ are the vertices of an isosceles triangle (two sides equal in length).

2.5 Equations of straight lines:

Any equation that can be written in the form,

$$Ax + By = C$$

Where we can't have both A and B be zero simultaneously is a line. It is okay if one of them is zero, we just can't have both be zero. Note that this is sometimes called the **standard form** of the line.

Given two points that are on the line we can graph the line and/or write down the equation of the line.

One of the more important ideas that we'll be discussing in this section is that of **slope**. The slope of a line is a measure of the *steepness* of a line and it can also be used to measure whether a line is increasing or decreasing as we move from left to right. Here is the precise definition of the slope of a line.

Definition: Given any two points on the line (x_1, y_1) and (x_2, y_2), the slope of the line is given by,

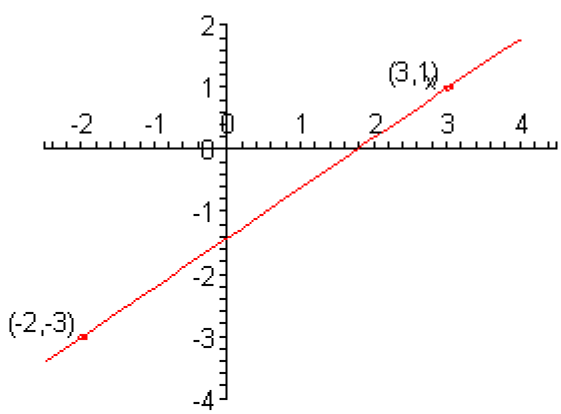
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

You will often hear the slope as being defined as follows,

$$m = \frac{rise}{run}$$

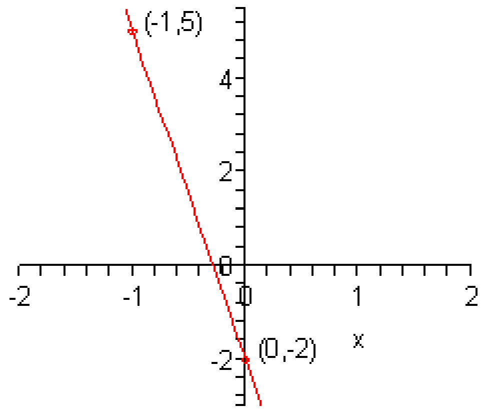
Slope can be either positive or negative

1) Positive slope: the line **RISES** to the right.

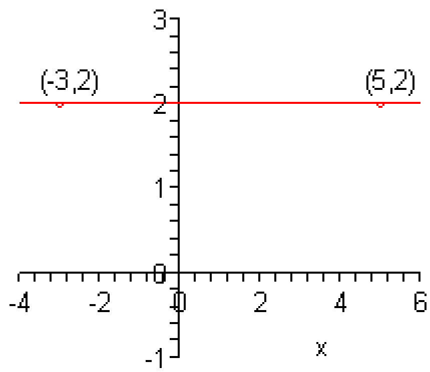


Notice that this line increases as we move from left to right.

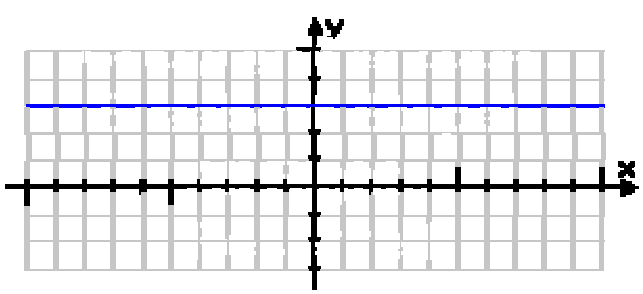
2) Negative slope: the line **FALLS** to the right.



Notice that this line decreases as we move from left to right.
** Two extreme cases regarding steepness are *horizontal and vertical* lines.
A **horizontal line has slope 0**, because the numerator $y_2 - y_1$ in the slope formula is 0.

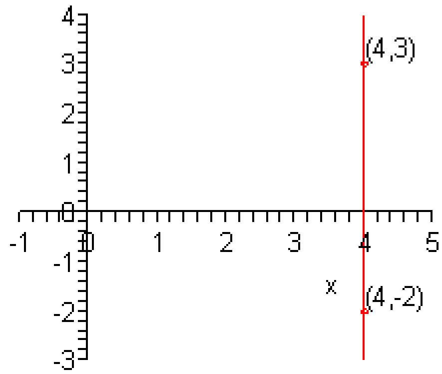


In this case we have a horizontal line.
Pick two points on the line and find the slope.
The equation of a horizontal line is: $y = c$ **, where c is a constant (number).**
EX 1: Graph $y = 3$



NOTE: Any time you have a "*y* equals a number" equation, with no *x* in it, the graph will always be a horizontal line.

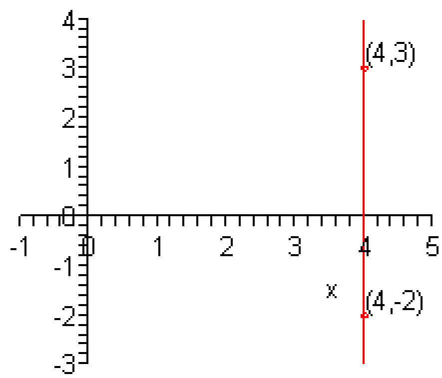
For a **vertical line, the slope is not defined** because the denominator $x_2 - x_1$ in the slope formula is 0.



In this case we have a vertical line.
Pick two points on the line and find the slope.

The equation of a vertical line is: $x = c$ **, where c is a constant (number).**

EX 2: Graph $x = 4$.



NOTE: Any time you have an "*x* equals a number" equation, with no *y* in it, the graph will always be a vertical line like this.

- **Graphing Equations of Straight lines:**
  The graph of a linear equation in two variables is a line. To graph a linear equation in two variables, find two points then draw the line connecting the two points. The easiest points are the *x-intercept* and the *y-intercept.*

  To find an *x-intercept*, let *y* =0 and solve for *x*.

  To find a *y-intercept*, let *x* =0 and solve for *y*.

- **Forms of Equations of Straight lines:**

1) Point-slope form:

$$y - y_1 = m(x - x_1)$$

Where *m* is the slope of the line and (x_1, y_1) is a point on the line.

EX 3: Suppose a line *L* passes through point $P_1(2,-3)$ and has slope $m = 3$.
Find a point-slope equation of line *L*.

EX 4: Find an equation of the line passing through the points $P_1(-4,-4)$ and $P_2(-5,2)$.

2) Slope-Intercept form:

$$y = mx + b$$

Where *m* is the slope of the line and $(0, b)$ is the *y-intercept.*

EX 5: Suppose a line *L* passes through point $P_1(2,-3)$ and has slope $m = 3$.
Find the slope-intercept equation of line *L*.

EX 6: Find the slope and *y-intercept* of $-3x+2y=-8$.

Note: When asked to find the slope and *y-intercept,* write the equation in slope-intercept form.

3) General form:

$$Ax + By = C$$

EX: Graph the line *L: 3x-5y=15*.

NOTE: To graph any line you have to find:

1. slope
2. *y-intercept (0,)*
3. *x-intercept (, 0)*

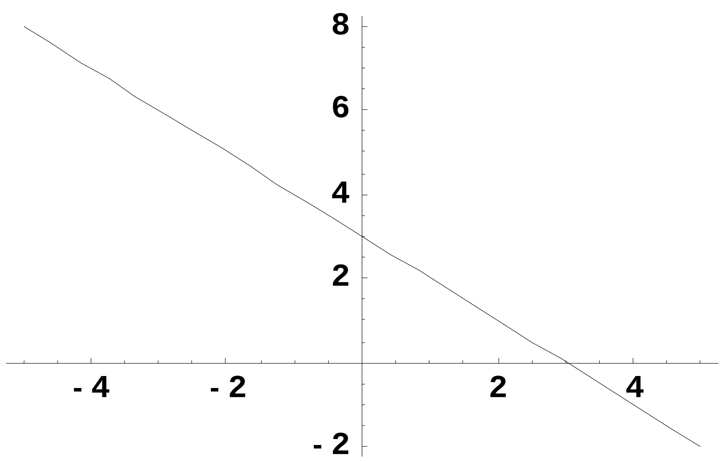
Now try the following problems:

1) Suppose a line L passes through point $P_1(2,-3)$ and has slope $m = 3$.
   a) Find the slope-intercept equation of line L.
   b) Find a point-slope equation of line L.

2) Find an equation of the line passing through the points $P_1(-4,-4)$ and $P_2(-5,2)$.

3) The equation of a vertical line is of the form ________________. The equation of a horizontal line is of the form _________________.

4) What is an equation of the line whose graph is

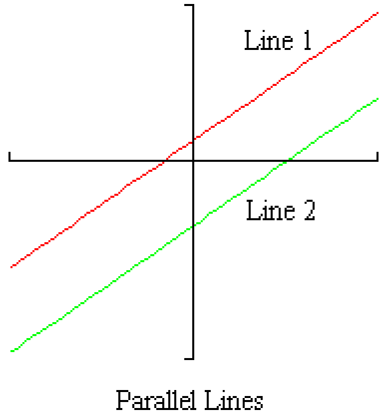


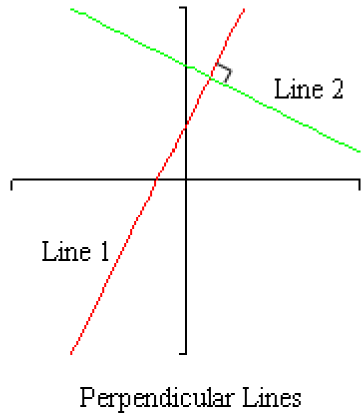
5) Graph the lines $y = 2x - 3$, $y = -2x + 3$, $y = -4$, and $x = 2$.

Please check HW problems as well.

2.6 Solving systems of linear equations with two variables:

* Parallel lines and Perpendicular lines:





Suppose that the slope of Line 1 is m_1 and the slope of Line 2 is m_2 We can relate the Slopes of parallel lines and slopes of perpendicular lines as follows:

Parallel: $m_1 = m_2$.

Perpendicular: $m_1.m_2 = -1$ **or** $m_2 = \frac{-1}{m_1}$

Example 1: Determine whether the following pair of lines are parallel, perpendicular or neither. Be able to explain your reasoning.

$$4x - 2y = 3$$
$$-2x + y = 1$$

Example 2: Find an equation of the line *L* through the point (–2, 1) which is parallel to the line $2x - 5y = 7$.

Example 3: Find an equation of the line *L* through the point (–2, 1) which is perpendicular to the line $2x - 5y = 7$

* SOLVING SYSTEMS OF LINEAR EQUATIONS:

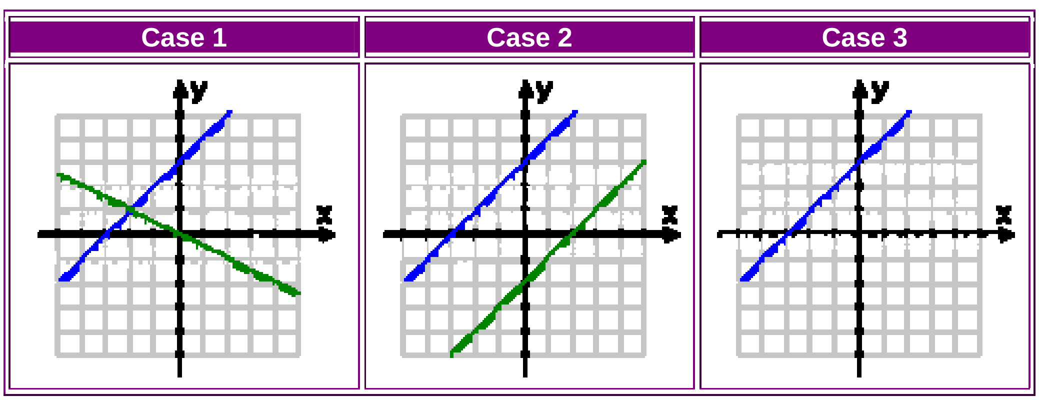
A system of two linear equations with two variables is any system that can be written in the form (called the standard form):

$$ax + by = p$$
$$cx + dy = q$$

BEFORE SOLVING THE SYSTEM YOU HAVE TO MAKE SURE THAT IT IS WRITTEN IN THIS FORM.

The solution of a system of linear equations will have one of the following properties:

1) **Exactly One solution,** the two lines meet **at exactly one point,** the point of intersection.
2) If a contradiction is obtained, such as 0=7 or 1=-1, then the system has **No solution**, the two lines never meet; they are **parallel**.
   Answer will be: **No Solution**. ϕ " empty set"
3) If an identity is obtained, such as 0=0, then the system has an **Infinite number of solutions,** the two lines **coincide (overlap).**
   Answer will be: **All Real Numbers**. $(-\infty, +\infty)$



We will be looking at **two methods for solving systems** in this section.

1) **The Substitution method**: In this method we will

-solve one of the equations for one of the variables and substitute this into the other equation.

-This will yield one equation with one variable that we can solve.

- Once this is solved, we substitute this value back into one of the equations to find the value of the remaining variable.

Example 4: Solve the following system.

$$\begin{aligned} 3x - y &= 7 \\ 2x + 3y &= 1 \end{aligned}$$

Solution:

(a) $\begin{aligned} 3x - y &= 7 \\ 2x + 3y &= 1 \end{aligned}$

The substitution method says that we need to solve one of the equations for one of the variables. Which equation we choose and which variable that we choose is up to you, but it is usually best to pick an equation and variable that will be easy to deal with. This means we should try to avoid fractions if at all possible.

In this case it looks like it will be really easy to solve the first equation for *y* so let us do that.

$$3x - 7 = y$$

Now, substitute this into the second equation.

$$2x + 3(3x - 7) = 1$$

This is an equation in *x* that we can solve.

$$\begin{aligned} 2x + +9x - 21 &= 1 \\ 11x &= 22 \\ x &= 2 \end{aligned}$$

This is the *x* portion of the solution.

Do NOT forget to go back and find the *y* portion of the solution. **This is one of the most common mistakes students make in solving systems.** To do this we can either plug the *x* value into one of the original equations and solve for *y*, or we can just plug it into our substitution that we found in the first step. Of course that will be easier to do.

$$y = 3x - 7 = 3(2) - 7 = -1$$

The solution is $x = 2$ and $y = -1$

2) **The Elimination method**:

- **Write both equations in the form A*x* + B*y* = C if needed.**
- If possible, Multiply one or both of the equations by appropriate numbers (*i.e.* multiply every term in the equation by the number) so that one of the variables will have the same coefficient with opposite signs.

- Then add the two equations together. Because one of the variables had the same coefficient with opposite signs it will be eliminated when we add the two equations.
- The result will be a single equation that we can solve for one of the variables. Once this is done substitute this answer back into one of the original equations.

Example 5: Solve each of the following systems of equations.

(a) $$\begin{aligned} 2x - 3y &= 0 \\ -4x + 3y &= -1 \end{aligned}$$

(b) $$\begin{aligned} 5x + 4y &= 1 \\ 3x - 6y &= 2 \end{aligned}$$

(c) $$\begin{aligned} 2x + 4y &= -10 \\ 6x + 3y &= 6 \end{aligned}$$

Solution:

(a) $$\begin{aligned} 2x - 3y &= 0 \\ -4x + 3y &= -1 \end{aligned}$$

Notice that both equations are in standard form. Since the *y* terms already have same coefficient and opposite signs let us work with these terms. When we add the two equations the *y* terms will be eliminated, and the resulting equation is

$$-2x = -1$$

Solving for x gives $x = \frac{-1}{-2} = \frac{1}{2}$

Since *x* is a fraction, if we plug this value into the first equation we will lose the fractions. Then solve for *y*.

$$\begin{aligned} 2\left(\frac{1}{2}\right) - 3y &= 0 \\ 1 - 3y &= 0 \\ -3y &= -1 \\ y &= \frac{1}{3} \end{aligned}$$

The solution is $x = \frac{1}{2}$ and $y = \frac{1}{3}$

You will find that if you plug $x = \frac{1}{2}$ and $y = \frac{1}{3}$ into BOTH equations of the original system, this is a solution to BOTH of them.

(b) $\begin{aligned} 5x + 4y &= 1 \\ 3x - 6y &= 2 \end{aligned}$

Here, we need to multiply one or both equations by constants so that one of the variables has the same coefficient with opposite signs. Since the *y* terms already have opposite signs let us work with these terms. It looks like if we multiply the first equation by 3 and the second equation by 2 the *y* terms will have coefficients of 12 and -12 which is what we need for this method.

$$\begin{aligned} 5x + 4y = 1 &\xrightarrow{\times 3} 15x + 12y = 3 \\ 3x - 6y = 2 &\xrightarrow{\times 2} \underline{6x - 12y = 4} \end{aligned}$$

$$21x = 7$$

Solving for x gives, $x = \frac{1}{3}$.

Again **do not forget to find *y***.

To find *y* we need to substitute the value of *x* into either of the original equations and solve for *y*. Since *x* is a fraction, if we plug this value into the second equation we will lose the fractions at least temporarily. Note that often this will not happen and we will be forced to deal with fractions whether we want to or not.

$$\begin{aligned} &3\left(\frac{1}{3}\right) - 6y = 2 \\ &1 - 6y = 2 \\ &-6y = 1 \\ &y = -\frac{1}{6} \end{aligned}$$

So, the solution is $x = \frac{1}{3}$ and $y = -\frac{1}{6}$

(c) $\begin{aligned} 2x + 4y &= -10 \\ 6x + 3y &= 6 \end{aligned}$

In this part all the variables are positive so we are going to have to force an opposite sign by multiplying by a negative number somewhere. Also, notice that in this case if we just multiply the first equation by -3 then the coefficients of the *x* will be -6 and 6.

Sometimes we only need to multiply one of the equations and can leave the other one alone. Here is this work for this part.

$$\begin{array}{lcr} 2x+4y=-10 & \xrightarrow{\times\ -3} & -6x-12y=30 \\ 6x+3y=6 & \xrightarrow{\text{same}} & \underline{6x+3y=6} \\ & & -9y=36 \\ & & y=-4 \end{array}$$

Finally, plug this into either of the equations and solve for *x*. We will use the first equation this time.

$$\begin{aligned} &2x+4(-4)=-10 \\ &2x-16=-10 \\ &2x=6 \\ &x=3 \end{aligned}$$

So, the solution to this system is $x=3$ and $y=-4$

Example 6: $\begin{aligned} 2x+3y&=6 \\ 4x+6y&=12 \end{aligned}$

Example 7: $\begin{aligned} x-3y&=1 \\ -2x+6y&=5 \end{aligned}$

We can also use systems of linear equations to solve word problems.

Example 8: An airplane flying with the wind can cover a certain distance in 2 hours. The return trip against the wind takes 2.5 hours. How fast is the plane and what is the speed of the wind, if the one-way distance is 600 miles.

Step 1: Understand the problem.

Make sure that you read the question carefully several times.

Since we are looking for two different rates (speeds), we will let

x **= rate/speed of the plane**

y **= the rate/speed of the wind**

Since this is a rate/distance problem, it might be good to organize our information using the distance formula.

Keep in mind that the wind speed is affecting the overall speed.

When the plane is with the wind, it will be going faster. That rate will be $x + y$.

When the plane is going against the wind, it will be going slower. That rate will be $x - y$.
Hint: You may use the following Chart to organize the given information:

	D=	R *	T
With the wind	600	$x + y$	2
Against the wind	600	$x - y$	2.5

Step 2: Devise a plan (translate).

Since we have two unknowns, we need to build a system with two equations.

Equation (1):

Rate times Time is equal to Distance

$$(x+y)(2) \qquad = \qquad 600$$

Equation (2):

Rate times Time is equal to Distance

$$(x-y)(2) \qquad = \qquad 600$$

Putting the two equations together in a system we get:

$2(x+y)=600$

$2.5(x-y)=600$

Step 3: Carry out the plan (solve).

This is a system of linear equations with two variables.

Simplify if needed.

We can simplify this by dividing both sides of equation (1) by 2 and equation (2) by 2.5 getting rid of the ( ) and decimals at the same time:

$$\frac{2(x+y)}{2} = \frac{600}{2}$$
$$\frac{2.5(x-y)}{2.5} = \frac{600}{2}$$

The new system:

$$x + y = 300$$
$$x - y = 240$$

At this point, you can use any method that you want to solve this system.

Multiply one or both equations by a number that will create opposite coefficients for either *x* or *y* if needed AND add the equations.

Since we already have opposite coefficients on *y*, we can go right into adding equations:

$$\begin{aligned} x + y &= 300 \\ \underline{x - y} &\underline{= 240} \\ 2x &= 540 \end{aligned}$$

Notice that since ***y*****'s have opposite coefficients, they dropped out.**

Solve for remaining variable. Solving for *x* we get:

$$\begin{aligned} 2x &= 540 \\ x &= 270 \end{aligned}$$

Solve for second variable. Using equation *x*+*y* = 300 to plug in 270 for *x* and solving for *y* we get:

$$x + y = 300$$
$$270 + y = 300$$
$$y = 30$$

Step 4: Look back (check and interpret).

You will find that if you plug x=270and y=30 into BOTH equations of the original system, this is a solution to BOTH of them.

Final Answer:

The airplane speed is 270 mph and the air speed is 30 mph

Another example,

Example 9: How many gallons of 20% alcohol solution and 50% alcohol solution must be mixed to get 9 gallons of 30% alcohol solution?

Step 1: Understand the problem.

Make sure that you read the question carefully several times.

Since we are looking for two different amounts, we will let

***x* = the number of gallons of 20% alcohol solution**

***y* = the number of gallons of 50% alcohol solution**

Step 2: Devise a plan (translate).

Let's use the following table to organize the given data.

	Part 1	Part 2	Mixture
Concentrate (alcohol) %	20%	50%	30%
Medium (gallons)	x	y	9
Amount of concentrate	$.2x$	$.5y$	.3(9)

Since we have two unknowns, we need to build a system with two equations.

Equation (1):

The total number of gallons is 9

$$x + y = 9$$

Equation (2):

Amount of alcohol in 20% solution	PLUS	Amount of alcohol in 50% solution	IS	Amount of alcohol in 30% solution

$$.2x \quad + \quad .5y \quad = \quad .3(9)$$

Putting the two equations together in a system we get:

$x + y = 9$

$.2x + .5y = 2.7$

Step 3: Carry out the plan (solve).

This is a system of linear equations with two variables.
We can simplify this by multiplying both sides of equation (2) by 10 and getting rid of the decimals:

$x + y = 9$

$10(.2x + .5y) = 10(2.7)$

$x + y = 9$

$2x + 5y = 27$

At this point, you can use any method that you want to solve this system. Let us use the **elimination method.**

Multiply one or both equations by a number that will create opposite coefficients for either *x* or *y* if needed AND add the equations.

If we multiply equation (1) by -2, then the *x*'s will have opposite coefficients.

Multiplying equations (1) by -2 and then adding that to equation (3) to eliminate the *x* terms we get:

$-2(x + y) = -2(9)$

$2x + 5y = 27$

$-2x - 2y = -18$

$2x + 5y = 27$

$3y = 9$

Solve for remaining variable. Solving for *y* we get:

$3y = 9$

$y = 3$

Solve for second variable. Using equation (1) to plug in 3 for *y* and solving for *x* we get:

$x + y = 9$

$x + 3 = 9$

$x = 6$

Step 4: Look back (check and interpret).
You will find that if you plug x=6 and y= 3 into BOTH equations of the original system, this is a solution to BOTH of them.

Final Answer:
6 gallons of 20% solution and 3 gallons of 50% solution

Now, try the following problems:

1. Determine whether the following pair of lines are parallel, perpendicular or neither. Be able to explain your reasoning.
$$\begin{aligned} 4x - 2y &= 3 \\ -2x + y &= 1 \end{aligned}$$

2. Find an equation of the line L through the point (–2, 1) which is parallel to the line $2x - 5y = 7$.

3. Find an equation of the line L through the point (–2, 1) which is perpendicular to the line $2x - 5y = 7$.

4. Solve the following systems of equations:

(a)
$$\begin{aligned} 2x - 3y &= 7 \\ 3x - \ \ y &= 1 \end{aligned}$$

(b)
$$\begin{aligned} 7x - 5y &= -1 \\ 3x + 2y &= 12 \end{aligned}$$

(c)
$$\begin{aligned} x + \ \ y &= 3 \\ 4x + 4y &= 9 \end{aligned}$$

2.7 Solving Systems of linear equations in three variables:

The general form of a linear equation in 3 variables is $Ax + By + Cz = D.$

BEFORE SOLVING THE SYSTEM YOU HAVE TO MAKE SURE THAT *EVERY EQUATION* IS WRITTEN IN THIS FORM.

The solution of a system of linear equations in 3 variables can be **no solution**, **one solution or infinite number of solutions.**

One Solution

If the system in three variables has one solution, it is an ordered triple (*x*, *y*, *z*) that is a solution to ALL THREE equations. In other words, when you plug in the values of the ordered triple, it makes ALL THREE equations TRUE.

No Solution

If the three planes are parallel to each other, they will never intersect. This means they do not have any points in common. In this situation, you would have no solution.

Infinite Solutions

If the three planes end up lying on top of each other, then there is an infinite number of solutions. In this situation, they would end up being the same plane, so any solution that would work in one equation is going to work in the other.

****The Reduction Method:**The method for solving a system of linear equations in three variables is similar to that used on systems of linear equations in two variables. We use the elimination to eliminate any variable, **reducing** the system to two equations in two variables.

Example 1: Solve the system

$x – 4y – z = 6$

$2x – y + 3z = 0$

$-3x + 2y – z = -4$

Solution: 1) First we start by numbering each equation for easy reference.

$x – 4y – z = 6$ Equation 1

$2x – y + 3z = 0$ Equation 2

$-3x + 2y – z = -4$ Equation 3

2) We choose two equations and use the elimination method to eliminate a variable. There are many ways to accomplish this, but we will eliminate *x* from equations 1 and 2:

$x - 4y - z = 6$	Equation 1
$-2(x - 4y - z) = -2(6)$ $-2x + 8y + 2z = -12$	Multiply both sides of Equation 1 by -2
$-2x + 8y + 2z = -12$ $\underline{2x - y + 3z = 0}$ $7y + 5z = -12$ Equation 4	Add the result from the previous step to Equation 2; we will call the result Equation 4

3) We must now eliminate the same variable, x, from another pair of equations. This time we will use equations 1 and 3:

$x - 4y - z = 6$	Equation 1
$3(x - 4y - z) = 3(6)$ $3x - 12y - 3z = 18$	Multiply both sides of Equation 1 by 3
$3x - 12y - 3z = 18$ $\underline{-3x + 2y - z = -4}$ $-10y - 4z = 14$ Equation 5	Add the result from the previous step to Equation 3; we will call the result Equation 5

4) We now have two equations, 4 and 5, which contain only the variables y and z. We will now consider these equations as a system of two equations in two variables:

$7y + 5z = -12$ $4(7y + 5z) = 4(-12)$ $28y + 20z = -48$	Multiply both sides of Equation 4 by 4
$-10y - 4z = 14$ $5(-10y - 4z) = 5(14)$ $-50y - 20z = 70$	Multiply both sides of Equation 5 by 5

$28y + 20z = -48$ $\underline{-50y - 20z = 70}$ $-22y = 22$ $y = -1$	Add the two equations obtained above; solve for *y*

5) Back-substitute $y = -1$ into either Equation 4 or 5 to find z:

$7y + 5z = -12$

$7(-1) + 5z = -12$

$-7 + 5z = -12$

$5z = -5$

$z = -1$

6) Back-substitute $y = -1$ and $z = -1$ into one of the three original equations to find x:

$x - 4y - z = 6$

$x - 4(-1) - (-1) = 6$

$x + 4 + 1 = 6$

$x + 5 = 6$

$x = 1$

The solution is $(1, -1, -1)$, the point of intersection.

*** RULES FOR SPECIAL CASES:**

When solving a system of linear equations:

1) If an identity is obtained, such as 0=0, then the system has an infinite number of solutions. Answer will be: **All Real Numbers**. $(-\infty, +\infty)$

2) If a contradiction is obtained, such as 0=7 or 1=-1, then the system has no solution. Answer will be: **No Solution**. ϕ " empty set"

Now, try the following problems:

1) Solve the following systems of equations:

a.

$$\begin{aligned} 2x + 5y - 4z &= 10 \\ 3x - 2y + \ z &= 3 \\ 4x + \ y - \ z &= 9 \end{aligned}$$

Answer: $x = 2, y = 2, z = 1$

b.

$$\begin{aligned} x + 2y + z &= 3 \\ 2x - y + 2z &= 6 \\ 3x + \ y - \ z &= 5 \end{aligned}$$

Answer: $x = 2, y = 0, z = 1$

c.

$$\begin{aligned} 3x + 2y + z &= 3 \\ x - 3y + \ z &= 4 \\ -\frac{3}{2}x - y - \frac{1}{2}z &= \frac{1}{4} \end{aligned}$$

Answer: No Solution