I.1. Let \( f : \mathbb{R} \to \mathbb{R} \) be continuous with asymptotes \( y = a_+x + b_+ \) as \( x \to +\infty \) and \( y = a_-x + b_- \) as \( x \to -\infty \) \((a_\pm, b_\pm \text{ some real numbers})\). Show that \( f \) is uniformly continuous.

I.2. Show that
\[
\lim_{r \to \infty} \int_0^r \frac{\sin t}{t} \, dt
\]
converges, but
\[
\lim_{r \to \infty} \int_0^r \frac{|\sin t|}{t} \, dt
\]
does not.

I.3. Let \( \{f_k\}_{k=1}^\infty \) be a sequence of continuous functions \((-1, 1) \to \mathbb{R}\) such that \( |f_k(x)| \leq M \), for all \( x \) and \( k \). Let \( g_k : (-1, 1) \to \mathbb{R} \) be defined by
\[
g_k(x) = \int_0^x f_k(t) \, dt.
\]
Show that \( \{g_k\}_{k=1}^\infty \) has a uniformly convergent subsequence.

I.4. Give an example of a sequence of functions \( f_n(x) \) in \([0, 1]\) such that \( \int_0^1 |f_n(x)| \, dx \to 0 \), as \( n \to \infty \), but \( f_n(x) \) does not converge to zero for any \( x \) in \([0, 1]\). Can you make the functions \( f_n \) continuous?
Part II. Do two of these problems.

II.1. Let \((X, \mathcal{M}, \mu)\) be a finite measure space. Let \(\{a_k\}_{k=1}^\infty\) be a strictly increasing sequence of positive numbers with \(c_0 a_k \leq a_{k+1} \leq c_1 a_k\) for some constants \(c_0, c_1\), both > 1. Suppose \(f : X \to \mathbb{R}\) is measurable. Show:

\[
\sum_{k=1}^{\infty} a_k \mu\{x : a_k \leq |f(x)|\} < \infty \iff f \in L^1(X, \mathcal{M}, \mu)
\]

II.2. Let \(\mathbb{R}[x]\) be the set of polynomials on \(\mathbb{R}\), let \(F = \{p(x)e^{-x^2/2} : p \in \mathbb{R}[x]\}\). Show that \(F\) is dense in \(L^2(\mathbb{R})\).

II.3. Let \(f(x, y), 0 \leq x, y \leq 1\), be a function such that for each \(x\), \(f(x, y)\) is an integrable function of \(y\), and \((\partial f(x, y)/\partial x)\) is a bounded function of \((x, y)\). Show that \((\partial f(x, y)/\partial x)\) is a measurable function of \(y\) for each \(x\) and

\[
\frac{d}{dx} \int_0^1 f(x, y)dy = \int_0^1 \frac{\partial}{\partial x} f(x, y)dy.
\]