Part I. (Do 3 problems)

1. Solve the Cauchy Problem

\[
\begin{align*}
\frac{\partial u}{\partial x_1} + 2x_2 \frac{\partial u}{\partial x_2} + \frac{\partial u}{\partial x_3} &= 3u \\
u(x_1, x_2, 0) &= f(x_1, x_2)
\end{align*}
\]

where \(f(x_1, x_2) \in C^1(\mathbb{R}^2)\).

2. A function \(f\) is radial if its value at \(x\) depends only on \(|x|\). Prove that a radial harmonic function on the ball \(B = \{x \in \mathbb{R}^n : |x| < 1\}\) is constant. Is this true if we replace \(B\) by the punctured ball \(\{x \in \mathbb{R}^n : 0 < |x| < 1\}\)? Justify your answer.

3. Solve the Cauchy problem

\[
\begin{align*}
u_{tt} - c^2 u_{xx} &= \cos x \\
u(x, 0) &= \sin x, \\
u_t(x, 0) &= x + 1.
\end{align*}
\]

4. Use the Fourier transform to prove that if \(f \in L^1(\mathbb{R}^n)\) satisfies \(f = f \ast f\), then \(f \equiv 0\).

Part II. (Do 2 problems)

1. Suppose \(u\) solves the equation \(\Delta u = 1\) in the unit ball \(x^2 + y^2 \leq 1\) of \(\mathbb{R}^2\) and satisfies \(u(x, 0) = u_y(x, 0) = 0\) for \(-1 < x < 1\). Find the polynomial \(p(x, y)\) of degree two satisfying

\[
u(x, y) = p(x, y) + o((x, y)^3)
\]

as \((x, y) \to (0, 0)\).

2. Let \(u \in C(\bar{\Omega}) \cap C^2(\Omega)\) solve the equation \(\Delta u + V \cdot Du = F\) in \(\Omega\) bounded domain, where \(V\) is a continuous vector field in \(\Omega\), \(F > 0\) in \(\Omega\), and \(u < 0\) on \(\partial \Omega\). Prove that \(u < 0\) in \(\Omega\).

3. Let \(U \subset \mathbb{R}^n\) be open and \(u \in W^{1,1}_{loc}(U)\). Show that \(|u| \in W^{1,1}_{loc}(U)\).