GRADUATE MATHEMATICS COURSES, FALL, 2016

Math 8007: Introduction to Methods in Applied Mathematics I
Prof. I. Klapper

Modeling and understanding our world through mathematical description and analysis are sources and inspiration for much of mathematics, and they complement theoretical and practical developments in other areas of science. The topics covered in this course introduce students to advanced mathematical tools and fundamental principles underlying many physical, biological, and social phenomena. This is the first semester of a two semester general overview of applied mathematics. Topics to be covered include modeling natural phenomena and derivation of equations of continuum and quantum mechanics; distributions and weak derivatives with applications to the analysis of heterogeneous media; Sturm-Liouville theory; solution methods for linear and nonlinear PDE, such as scaling and dimensional analysis, using natural curvilinear coordinates, separation of variables, and Fourier and Laplace transforms.

Prerequisites: Undergraduate level Calculus III and Ordinary differential equations.

Textbooks: (a) Walter A. Strauss, Partial Differential Equations. An Introduction, John Wiley Sons. Inc., 1992, ISBN: 0-471-54868-5 (b) H.F. Weinberger, A First Course in Partial Differential Equations: with Complex Variables and Transform Methods, Dover Publications, 1995, ISBN-10: 048668640X, ISBN-13: 978-04866864001

Math 8011: Abstract Algebra I
Prof. Martin Lorenz

This course, the first semester of a year-long graduate-level introduction to abstract algebra, is roughly organized into three main parts: Groups (Chapters 1-6 of the textbook), Rings and Modules (Chapters 7-11), and Fields (Chapter 13). The indicated chapters contain too much material to be completely covered in the first semester; so a selection will be made.

The abstract algebra sequence Math 8011/8012 is a prerequisite for many of the higher-level graduate courses in pure mathematics and it provides the background needed for the PhD qualifying exam in Algebra.

Prerequisites: Math 3098 or equivalent or permission of instructor.

Textbook: Dummit & Foote, *Abstract Algebra*, 3rd ed., John Wiley & Sons, 2004.

Math 8023: Numerical Differential Equations I
Prof. B. Seibold

This course is designed for graduate students of all areas who are interested in numerical methods for differential equations, with focus on a rigorous mathematical basis. The first half of the course is devoted to efficient numerical methods for ordinary differential equations, including the topics: Runge-Kutta and multistep methods, truncation errors, adaptive time stepping, stability, stiff problems. The second half covers numerical methods for linear partial differential equations, including the topics: finite difference methods and finite element method for elliptic problems, error analysis, method of lines, heat equation, stability, wave propagation, modified equation.

Textbook: Randall J. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations — Steady State and Time Dependent Problems*, SIAM, 2007.

Prerequisite: Math 5043 and Math 5045 or permission of instructor.

Math 8031: Probability
Prof. W.-S. Yang

Course Goals: This course focuses on the rigorous foundation of graduate level probability theory.

Topics Covered: Probability measures, mathematical expectations, various convergence concepts, convergence theorems, the Law of Large Numbers, characteristic functions, the Central Limit Theorem, random walks and Markov chains.

Textbook: 1. K. L. Chung "A First Course in Probability", 3rd Edition Academic Press. 2. Richard Durrett "Probability: Theory and Examples". 4th Edition, Cambridge University Press.

Prerequisites: Math 8041 or permission by the instructor.

Math 8041: Real Analysis
Prof. W.-S. Yang

This course and its continuation Math 8042 cover the core areas of analysis: Lebesgue measure and integration theory, differentiation, abstract measures and integration, Hilbert spaces, and Hausdorff measure and fractals. Emphasis will be on exercises and problems.

Textbook: Measure and Integral, An Introduction to Real Analysis, by R. Wheeden and A. Zygmund, Marcel Dekker, 1977, ISBN: 0824764994.

Prerequisites: Basic knowledge of real variables and Euclidean topology, sequences of functions, Riemann integration.

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Math 8051: Functions of a Complex Variable I
Prof. S. Berhanu

The *Functions of a Complex Variable Sequence* Math 8051-8052 serves as an introduction to the theory of analytic functions of one complex variable. Complex Analysis of one variable provides useful background for various areas of mathematics including Harmonic Analysis, Partial Differential Equations, Number theory, Several Complex Variables, Algebraic Geometry and Differential Geometry.

Topics covered in the course include elementary properties and examples of holomorphic functions; differentiability and analyticity, the Cauchy-Riemann equations; power series; conformality; complex line integrals, the Cauchy Integral Formula and Cauchy's Theorem; applications of the Cauchy Integral Formula- power series expansion for a holomorphic function, the Maximum Modulus principle, the Cauchy estimates, Liouville's Theorem; Singularities of holomorphic functions, Laurent expansions, the calculus of residues and applications to the calculation of definite integrals and sums; zeros of a holomorphic function, the Argument Principle, Rouche's Theorem, Hurwitz's Theorem; conformal mappings and harmonic functions.

Textbook: John B. Conway, *Functions of One Complex Variable*, Springer-Verlag

Additional references:

(1) Robert E. Greene and Steven G. Krantz, *Function Theory of One Complex Variable*, Wiley Interscience
(2) Lars V. Ahlfors, *Complex Analysis*, McGraw-Hill
(3) Elias M. Stein and Rami Shakarchi , *Princeton Lectures in Analysis II, Complex Analysis*, Princeton University Press
(4) E. Freitag and R. Busam, *Complex Analysis*, Universitext.

Prerequisite: Calculus of several variables and an undergraduate level course in complex variables.

8061. Differential Geometry and Topology I
Prof. V. Dolgushev

This course is a year-long primer on differentiable manifolds and their topology; it provides the background needed for the PhD qualifying exam in Differential Geometry Topology. The material of the first semester (Math 8061) includes the elementary theory of smooth manifolds, Morse-Sard theorem, oriented intersection theory, and integration on manifolds. The material of the second semester (Math 8062) includes the fundamental group, van Kampen's theorem, covering spaces, homology and cohomology. Time permitting, the course will also cover one or both of Poincaré duality and the rudiments of Hodge theory.

Textbooks: V. Guillemin and A. Pollack, *Differential Topology*, AMS Chelsea Publishing, A. Hatcher, *Algebraic Topology*, Cambridge University Press.

Math 8141: Partial Differential Equations I
Prof. B. Rider

A partial differential equation (PDE) is an equation involving functions and their partial derivatives. Since many natural laws can be expressed in terms of rates of change, PDEs appear and have applications to an enormous number of questions. For example, PDEs describe the propagation of sound and heat, the motion of fluids, the behavior of electric and magnetic fields, and the behavior of financial markets. We will start the course with first order equations. Next we will focus on three second order equations that contain the ideas and the germs of generality to study more general PDEs: the Laplace equation, the heat equation, and the wave equation. The solutions to these three equations have different qualitative and quantitative properties and their study is essential to understand elliptic, parabolic and hyperbolic equations. .

The course will be useful for students in analysis, geometry, applied mathematics, and engineering. It will also prepare students to take the qualifying exam in partial differential equations, see syllabus for this exam at `https://math.temple.edu/~gutierre/math8142/topics.for.pde.comp.exam.pdf`.

Prerequisites: Advanced calculus.

Textbooks:

(1) L. C. Evans, *Partial Differential Equations*, Graduate Texts in Mathematics vol. 19, American Mathematical Society, 1998, ISBN: 0-8218-0772-2.
(2) D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, ISBN: 9783540411604.
(3) F. John, *Partial Differential Equations*, Springer, 1982, 4th edition, ISBN: 3-540-90609-6.

Math 9120: Mapping Class Groups

Prof. D. Futer

This course will focus on the symmetry groups of manifolds and cell complexes. We will look at both the geometric and algebraic properties of the *mapping class group*, which can roughly be thought of as the group of symmetries of a surface. The hyperbolic geometry of surfaces will play a major role in this investigation. We will also get a lot of information about the mapping class group by considering its action on the *complex of curves*, which is a combinatorial object that records the intersection pattern of simple closed curves. Throughout the semester, we will pass back and forth between geometry and algebra.

Textbook: Benson Farb and Dan Margalit, *A primer on mapping class groups*, Princeton University Press.

Prerequisite: Math 8061-62.

Math 9420: Nonlinear pdes and applications
Prof. Cristian E. Gutiérrez

Fully nonlinear pdes appear in several areas within Mathematics and in applications in broader scientific disciplines such as fluid dynamics, phase transitions, mathematical finance, geometric optics, and image processing in computer science. In the past few decades, there have been many new developments in this area including the understanding of regularity of generalized solutions, the study of singularities and symmetric properties of solutions. A goal in this course is to present some of these important developments including an introduction to Monge-Ampère (MA) type equations and its applications to optimal mass transport and geometric optics. These are, in general, equations involving the Jacobian determinant of a map, and arise in the mathematical description of optical, acoustic, and electromagnetic applications, in particular, in lens and reflector antenna design.

The course will present basic facts about the MA equation such us weak solutions, existence, uniqueness and describe some regularity results. We next build up on these ideas to show how to construct generalized solutions to other problems by means of the Minkowski method from convex geometry and by optimal mass transportation. The physical background underlying some of these problems is related to Maxwell's equations that will be described in detail. Other topics might be described depending on the interest of the students. If there are enough interested students the course will continue in the Spring 2015.

The course would be useful for graduate students interested in analysis, applied mathematics, physics and engineering.

The prerequisites are knowledge of real analysis and basic pdes.

References

[1] M. Born and E. Wolf. *Principles of Optics, Electromagnetic theory, propagation, interference and diffraction of light*. Cambridge University Press, seventh (expanded), 2006 edition, 1959.

[2] D. Gilbarg and N. S. Trudinger *Elliptic Partial Differential Equations of Second Order*. Springer-Verlag, second edition, revised 3rd printing 1998.

[3] D. Marcuse *Light Transmission Optics.* Van Nostrand, 2nd Edition, 1982.

[4] C. Villani. *Optimal transport, old and new*. Optimal transport, old and new. Grundlehren der mathematischen Wissenschaften, Vol.338, Springer-Verlag, 2009. Available for download at `http://cedricvillani.org/for-mathematicians/surveys-books/`

[5] C. E. Gutiérrez. *The Monge–Ampère equation*. Birkhäuser, Boston, MA, 2nd edition, 2016.