GRADUATE MATHEMATICS COURSES, FALL 2022

Math 5043: Introduction to Numerical Analysis
MW 5:10-6:30
Prof. D. Szyld

During the first semester of this course, the student is introduced to basic concepts in numerical analysis and scientific computing. In this discipline, algorithms for the solution of specific problems arising in science and engineering using computers, are presented and analyzed. The goal is to learn algorithms which approximate the solutions, in other words, one wants to guarantee that the answer produced by the computer code resembles the true solution. At the same time, one wants to produce algorithms which converge to the solution in a reasonable amount of time.

Some of the specific methods which will be studied include: Finding roots of non-linear equations. Approximation and interpolation of functions. Numerical integration. In addition, we will study how computers store and manipulate data, so we can study how errors produced by the use of non-exact arithmetic are generated and propagated in the specific algorithms. Stability of the algorithms will be studied as well.

Textbook: J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Third Edition, Springer, 2010.

Math 5057: Introduction to Methods in Applied Mathematics I
TR 3:30-4:50
Prof. Y. Grabovsky

This course provides the student with the toolbox of an applied mathematician: derivation of PDE of continuum mechanics; solution methods in special domains; scaling and dimensional analysis; PDEs in polar, spherical, and cylindrical coordinates; weak solutions of PDEs for modeling heterogeneous media and shock waves; calculus of variations and analytical mechanics; control theory; dynamical systems and bifurcations; asymptotic analysis of integrals, ODEs and PDEs. This course prepares students for the PhD qualifying exam in Methods of Applied Mathematics, as well as for research in Applied Mathematics and Analysis.

Math 8011: Abstract Algebra I
MW 10:30-11:50
Prof. M. Stover

This course, the first part of a two-semester sequence, gives an introduction to the terminology and methods of modern abstract algebra. The material to be covered in the fall semester is roughly organized into three main parts: Groups (Chapters 1 – 6 in the textbook), Rings and Modules (Chapters 7 – 11), and Fields (Chapter 13). The indicated chapters contain far too much material to be completely covered in one semester; so a selection will be made. The course and its sequel (Math 8012) are prerequisites for many of the higher-level graduate courses, and together they provide the background needed for the PhD qualifying exam in Algebra.

Prerequisites: Math 3098 or equivalent or permission of instructor.
Textbook: Dummit & Foote, *Abstract Algebra*, 3rd ed., John Wiley & Sons, 2004.

Math 8031: Probability Theory
MW 10:30-11:50
Prof. A. Yilmaz

In this course, we will introduce the axioms and fundamental notions of probability based on measure and integration, develop the theory with the accompanying probabilistic intuition, which is equally important, formulate and prove the strong law of large numbers for independent random variables, define and characterize weak convergence of probability measures, and give a rigorous treatment of the central limit theorem.

Prerequisites: You should be comfortable with Advanced Calculus and undergraduate-level Real Analysis. Any previous exposure to undergraduate-level Probability Theory or graduate-level Real Analysis would be helpful, but the course will be self-contained in those regards, i.e., they are not prerequisites.

Textbook: L. B. Koralov and Y. G. Sinai, Theory of Probability and Random Processes, 2nd Ed., Springer, 2007 (corrected 2nd printing 2012). We will cover Chapters 1–4 and 7–10 from Part I of the book.

Sequel: In the Spring semester, we will offer a course on Stochastic Processes, based mostly on Part II of the book.

Math 8041: Real Analysis I
MW 12:30-1:50
Prof. I. Mitrea

The year-long sequence 8041–8042 covers the core areas of analysis and it focuses on the following fundamental topics:

- **Topological vector spaces** (open and closed sets, separation properties, Uryshon's Lemma);
- **Abstract measure theory** (abstract outer measures and measures, sigma algebras, singular measures, completion and regularity of measures, absolute continuity, Caratheodory's measurability criterion, Caratheodory's construction of a measure from an outer measure, Lebesgue spaces, Hölder's, Minkowski's and Jensen's inequalities, completeness, pointwise convergence, Lebesgue's Monotone and Dominated Convergence Theorems, Fatou's Lemma, integration of series of functions, convergence in measure, Lusin's and Egorov's Theorems, integration on product spaces, Fubini's and Tonelli's Theorems, Radon-Nikodym's Theorem, duals of Lebesgue spaces);
- **The Lebesgue measure** (construction via the Riesz representation Theorem, integration, change of variables, the role of sets of Lebesgue measure zero, connections with the Riemann integral, Vitali's example of a set which is not Lebesgue measurable);
- **Hilbert spaces** (inner product spaces, orthogonality, completeness, duals, basis, Parseval's inequality);
- **Hausdorff measure and fractals** (the Hausdorff outer measure and its properties, Radon measures and outer measures, Hausdorff dimension, construction and properties of the Van Koch snowflake, the ternary Cantor set, the 4 point Cantor dust set)

Possible Textbooks:

- W. Rudin, Real and Complex Analysis (International Series in Pure and Applied Mathematics)
- G. B. Folland, Real Analysis: Modern Techniques and Their Applications (Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts)

Additional references:

- L. C. Evans and R. F. Gariepy, Measure Theory and Fine Properties of Functions (Studies in Advanced Mathematics)
- M. Spivak, Calculus on Manifolds (ebook)
- Measure and Integral, An Introduction to Real Analysis, by R. Wheeden and A. Zygmund, Marcel Dekker, 1977, ISBN: 0824764994.
- Real Analysis : Measure Theory, Integration, and Hilbert Spaces (Princeton Lectures in Analysis III) by Elias M. Stein and Rami Shakarchi, Princeton University Press (2005), ISBN: 0691113866.

Prerequisites: Basic knowledge of real variables and Euclidean topology, sequences of functions, Riemann integration.

Math 8051: Functions of a Complex Variable I
TR 12:30-1:50
Prof. G. Mendoza

This is the first semester of a year-long course. Topics for the first semester include: Elementary properties and examples of holomorphic functions; differentiability and analyticity, the Cauchy-Riemann equations; power series; conformality; complex line integrals, the Cauchy integral formula, Cauchy's theorem and applications; power series expansion of holomorphic functions, the Maximum Modulus Principle; Liouville's Theorem; singularities of holomorphic functions, Laurent expansions, the calculus of residues and applications to the calculation of definite integrals and sums; zeros of a holomorphic function, the Argument Principle, Rouche's Theorem, Hurwitz's Theorem; conformal mappings.

Topics for the second semester include harmonic functions, the Poisson integral formula, maximum and minimum principles, the mean value property, the Dirichlet problem, Harnack's inequality; spaces of holomorphic and meromorphic functions, the Riemann Mapping Theorem; analytic continuation; Weierstrass and Hadamard's Factorization Theorems; Picard's Theorems; introduction to Riemann Surfaces.

Textbook: John B. Conway, *Functions of One Complex Variable*, Springer.

Math 8061: Differential Geometry & Topology I
TR 9:30–10:50
Prof. S. Taylor

This is an introduction to the basic theory of smooth manifolds that will prepare students for that portion of the PhD qualifying exam in geometry and topology. Topics include smooth manifolds and maps, transversality, intersection theory, vectors fields, Euler characteristic, and differential forms. If there is time, we will cover additional topics like vector bundles, Morse theory, and classification of compact 1- and 2-dimensional manifolds.

Prerequisites: Abstract Algebra (Math 8011). Abstract Algebra can be taken concurrently.

Textbook: Lee, *Introduction to Smooth Manifolds*.

Math 8200: Topics in Applied Mathematics: Math Biology I
TR 9:30-10:50
Prof. I. Klapper

This course is Part I of an introduction to and survey of basic ideas of mathematical biology for graduate students including those already engaged in dissertation research in related subjects. We will move from temporal dynamics to spatio-temporal models, covering population and ecological models as well as transport and other physical processes (e.g., pattern formation) in biological systems. This course will serve as a prerequisite for Part II, to be offered in Spring 2023 (by Dr. Queisser), covering more targeted topics including neuron physiology and activity.

Prerequisites: Previous experience with PDE and some physics is helpful but not required.

Math 9041: Functional Analysis I
TR 11-12:20
Prof. C. Gutierrez

This is a course covering fundamental topics of functional analysis having multiple applications in theoretical and applied mathematics areas such us harmonic analysis, pdes, probability, optimization, economics, physics, etc. It will cover the following:

(1) The Hahn-Banach theorem and applications
(2) The uniform boundedness principle, the closed graph, and open mapping theorems
(3) Unbounded linear operators basics
(4) Weak topologies, duality
(5) Compact operators and spectral decomposition
(6) Applications to boundary value problems
(7) Fourier transform and distributions (time permitting).

Textbook (containing a very large number of exercises and solutions): Haim Brezis, ”Functional Analysis, Sobolev spaces and partial differential equations”, Springer, 2010, ISBN: 978-0-387-70913-0.

Additional (classical) references:

- W. Rudin, ”Functional Analysis”, 1991, ISBN-13: 978-0070542365.
- F. Riesz and B. Nagy, ”Functional Analysis”, Dover, 1990.
- Banach S. Theory of Linear Operations. Volume 38, North-Holland Mathematical Library, 1987, ISBN 0-444-70184-2.
- Dunford, N. and Schwartz, J.T.: Linear Operators, General Theory, John Wiley & Sons, and other 3 volumes, includes visualization charts.

Prerequisites: Good knowledge of real analysis at the level of the books by Wheeden and Zygmund, Folland, or Royden.

Math 9100: Topics in Algebra: Introduction to Lie algebras with applications
MW 9:00-10:20
Prof. V. Dolgushev

The material of this one-semester course can be divided into two parts. The first part of the semester will be devoted the standard toolbox of Lie theory. This toolbox includes: the universal enveloping algebra of a Lie algebra, PBW theorem, free Lie algebras, nilpotent, solvable and semi-simple Lie algebras. In the first part of the course, we will also touch on some aspects of representation theory of Lie algebras. The second part of the course is devoted to finite type invariants of knots. In this part, we will introduce chord diagrams, open and closed Jacobi diagrams and describe ”links” between these objects. At the end of the course, I will show how Lie algebras can be used to study the objects related to finite type invariants of knots. If time permits, we will have sessions about Python packages for symbolic computing for Lie algebras and objects related to finite type invariants of knots.

Textbooks:

- S. Chmutov, S. Duzhin, J. Mostovoy, ”Introduction to Vassiliev Knot Invariants”,
- J.E. Humphreys ”Introduction to Lie Algebras and Representation Theory”;
- J.P. Serre ”Lie Algebras and Lie Groups”.

Prerequisites: Math 8011 and 8012.

Math 9200: Topics in Numerical Analysis I — Computational Methods for Flow Problems
TR 2:00–3:30
Prof. B. Seibold

Course Description: This course provides an overview of numerical methods for many important types of flow problems, ranging from incompressible fluids (Navier-Stokes equations), over shock problems (such as the compressible Euler equations), front propagation problems (such as multiphase flows), kinetic equations (Boltzmann equation, radiative transfer), to network flows (such as traffic on roads).

One third of the course will be devoted to the modeling, derivation, and mathematical/physical properties of the equations and their solutions; and two thirds to the design of efficient and robust numerical methods for their solution. The computational approaches include: finite volume methods, finite difference methods, meshfree and particle methods, level set methods, immersed interface and phase field models, moment methods.

The goal of this course is provide a broad perspective on these important types of flow problems, their connections, and how to tackle them computationally. This course provides the needed familiarity with each topic to enable the participants to engage into further studies via literature.

Textbooks: TBA.

Prerequisites: Solid familiarity with multivariable calculus; knowledge of numerical analysis; knowledge of ordinary and partial differential equations; programming experience.

Math 9500 Topics in Geometry & Topology: 3-manifold topology
TR 11-12:20
Prof. D. Futer

This course will survey several topics in the topological and geometrical study of 3-dimensional manifolds. We will study several different constructions of 3-manifolds: triangulations, Heegaard splittings (gluing handlebodies together), Dehn surgery, and mapping tori. Each of these constructions provides the resulting space with some structure, and we will study that structure. We will also survey the theory of incompressible surfaces in 3-manifolds, including the sphere and torus decomposition that canonically cut a manifold into geometric pieces. If time permits, we will discuss finite covers in 3-manifolds (and the attendant group theory), as well as algorithmic questions.