

GRADUATE MATHEMATICS COURSES, FALL 2021

Math 5043: Introduction to Numerical Analysis **TR 2:00-3:20** **Prof. G. Queisser**

During the first semester of this course, the student is introduced to basic concepts in numerical analysis and scientific computing. In this discipline, algorithms for the solution of specific problems arising in science and engineering using computers, are presented and analyzed. The goal is to learn algorithms which approximate the solutions, in other words, one wants to guarantee that the answer produced by the computer code resembles the true solution. At the same time, one wants to produce algorithms which converge to the solution in a reasonable amount of time.

Some of the specific methods which will be studied include: Finding roots of non-linear equations. Approximation and interpolation of functions. Numerical integration. In addition, we will study how computers store and manipulate data, so we can study how errors produced by the use of non-exact arithmetic are generated and propagated in the specific algorithms. Stability of the algorithms will be studied as well.

Textbook: J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Third Edition, Springer, 2010.

Math 8007: Introduction to Methods in Applied Mathematics I **TR 3:30-4:50** **Prof. Y. Grabovsky**

This course provides the student with the toolbox of an applied mathematician: derivation of PDE of continuum mechanics; solution methods in special domains; scaling and dimensional analysis; PDEs in polar, spherical, and cylindrical coordinates; weak solutions of PDEs for modeling heterogeneous media and shock waves; calculus of variations and analytical mechanics; control theory; dynamical systems and bifurcations; asymptotic analysis of integrals, ODEs and PDEs. This course prepares students for the PhD qualifying exam in Methods of Applied Mathematics, as well as for research in Applied Mathematics and Analysis.

Math 8011: Abstract Algebra I **TR 11:00-12:20** **Prof. J. Lang**

This course, the first part of a two-semester sequence, gives an introduction to the terminology and methods of modern abstract algebra. The material to be covered in the fall semester is roughly organized into three main parts: Groups (Chapters 1 – 6 in the textbook), Rings and Modules (Chapters 7 – 11), and Fields (Chapter 13). The indicated chapters contain far too much material to be completely covered in one semester; so a selection will be made. The course and its sequel (Math 8012) are prerequisites for many of the higher-level graduate courses, and together they provide the background needed for the PhD qualifying exam in Algebra.

Prerequisites: Math 3098 or equivalent or permission of instructor.

Textbook: Dummit & Foote, *Abstract Algebra*, 3rd ed., John Wiley & Sons, 2004.

Math 8013: Numerical Linear Algebra I
MW 5:10-6:30
Prof. D. Szyld

This introductory graduate course is directed to any student interested in applied mathematics or applied science in general. It concentrates on the study of the solution of linear systems of algebraic equations and of eigenvalue problems. These topics are at the core of many problems in science and engineering, especially the solution of differential equations. Students learn to solve these problems, to do some computational mathematics (fundamental in today's scientific world), and to analyze each possible method with a critical mind. Topics to be covered include: matrix factorizations and their applications. Conditioning and stability issues. Iterative methods.

Prerequisites: working knowledge of a computer language such as MATLAB, undergraduate linear algebra (matrix analysis, subspaces, etc. – these will be reviewed during the first week of classes).

Textbook: L.N. Trefethen and D. Bau, III, *Numerical Linear Algebra*, Society of Industrial and Applied Mathematics, 1997

Math 8023: Numerical Differential Equations I
TR 11:00–12:20
Prof. B. Seibold

This course focuses on numerical methods for ordinary and partial differential equations. Some of the topics covered are fundamentals of numerical methods such as stability, convergence and error analysis, various numerical approaches such as finite difference, finite elements and spectral methods applied to boundary value problems, advection, diffusion problems, wave propagation problems and flow and interface problems.

Textbook: Randall J. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations - Steady State and Time Dependent Problems*, SIAM, 2007.

Prerequisites: Math 5043 and Math 5045 or permission of instructor.

Math 8041: Real Analysis I
MW 10:30-11:50
Prof. M. Ignatova

This course covers the classical theory of Lebesgue integral and its applications. Closely related topics such as functions of bounded variation, the Riemann-Stieltjes integral, Fubini's theorem, and L^p -classes will also be discussed.

Prerequisites: Math 5041 or equivalent.

Textbook: Richard Wheeden and Antoni Zygmund, *Measure and Integral: An Introduction to Real Analysis* (Pure and Applied Mathematics) [Hardcover], CRC Press; 1st edition (1977), ISBN-10: 0824764994, ISBN-13: 978-0824764999.

Math 8051: Functions of a Complex Variable I
TR 12:30-1:50
Prof. S. Berhanu

This is the first semester of a year-long course. Topics for the first semester include: Elementary properties and examples of holomorphic functions; differentiability and analyticity, the Cauchy-Riemann equations; power series; conformality; complex line integrals, the Cauchy integral formula, Cauchy's theorem and applications; power series expansion of holomorphic functions, the Maximum Modulus Principle; Liouville's Theorem; singularities of holomorphic functions, Laurent expansions, the calculus of residues and applications to the calculation of definite integrals and sums; zeros of a holomorphic function, the Argument Principle, Rouché's Theorem, Hurwitz's Theorem; conformal mappings.

Topics for the second semester include harmonic functions, the Poisson integral formula, maximum and minimum principles, the mean value property, the Dirichlet problem, Harnack's inequality; spaces of holomorphic and meromorphic functions, the Riemann Mapping Theorem; analytic continuation; Weierstrass and Hadamard's Factorization Theorems; Picard's Theorems; introduction to Riemann Surfaces.

Prerequisites: Math 4051 or equivalent or permission of instructor.

Textbook: John B. Conway, *Functions of One Complex Variable*, Springer.

Math 8061: Differential Geometry & Topology I
TR 9:30-10:50
Prof. D. Futer

This is an introduction to the basic theory of smooth manifolds that will prepare students for that portion of the PhD qualifying exam in geometry and topology. Topics include smooth manifolds and maps, transversality, intersection theory, vector fields, Euler characteristic, and differential forms. If there is time, we will cover additional topics like vector bundles, Morse theory, and classification of compact 1- and 2-dimensional manifolds.

Prerequisites: Concepts of Analysis (Math 5041-5042) or equivalent and Abstract Algebra (Math 8011). Abstract Algebra can be taken concurrently.

Textbook: Lee, *Introduction to Smooth Manifolds*.

Math 8141: Partial Differential Equations I
MW 1:00-2:20
Prof. C. Gutierrez

A partial differential equation (PDE) is an equation that expresses a relation between a function and its partial derivatives. Since many processes (physical, chemical, etc) can be expressed in terms of rates of changes, PDEs appear and have applications to an enormous number of questions. For example, PDEs describe the propagation of sound and heat, the motion of fluids, the behavior of electric and magnetic fields, and the behavior of financial markets. PDEs are also crucial in understanding and solving various geometric problems.

The first semester course is intended to provide the student a basic introduction to the subject, including first-order PDEs and the three second order equations that arise in mathematical physics: the Laplace equation, the heat equation, and the wave equation. The solutions of these equations have different qualitative and quantitative properties and their study is essential for understanding the more general elliptic, parabolic and hyperbolic equations which will be the subject of the second semester course. The Fourier transform and Sobolev spaces and their applications to PDEs will be introduced and developed during the two semesters.

The course will be useful for students in analysis, applied mathematics, geometry, physics, and engineering.

Prerequisites: Basic concepts of real analysis; advanced calculus of several variables; knowledge of Lebesgue integration is useful.

Textbooks:

- (1) L. C. Evans, *Partial Differential Equations*, Graduate Texts in Mathematics, vol. 19, American Mathematical Society, 1998, ISBN: 0-8218-0772-2.
- (2) D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, ISBN: 9783540411604.

Math 9073: Geometric Group Theory
TR 2:00–3:20
Prof. S. Taylor

This semester-long course will survey the rapidly expanding field of geometric group theory, focusing on the role played by negative curvature. We will begin with classical combinatorial techniques used to construct and study infinite discrete groups. After introducing basic concepts in coarse geometry, we will turn our attention to Gromov's notion of hyperbolic groups. In addition to studying geometric, algebraic, and algorithmic properties of these groups, we will keep an eye towards several generalizations of hyperbolicity that have recently played a large role in understanding many geometrically significant groups. As examples, we will also touch on the study of mapping class groups, outer automorphism groups of free groups, and cubical groups.

Prerequisites: Math 8061-62.

Math 9071: Differential Topology: The topology of fiber bundles
MW 9:00-10:20
Prof. G. Mendoza

The course will begin with an introductory lectures on general facts about fiber bundles, but will focus primarily on topological aspects of vector bundles. Topics include Chern classes (and obstructions to triviality), connections, an introduction to Lie groups as needed for the study of principal G -bundles, groups of bundle isomorphism of a given vector bundle, special structures of the tangent bundle (involutive structures such as complex and CR structures). Time permitting, there will be an introduction to K -theory and the Atiyah-Singer index theorem.

References:

- (1) Kobayashi: Differential Geometry of Complex Vector Bundles.
- (2) Kobayashi: Transformation Groups in Differential Geometry
- (3) Milnor & Stasheff: Characteristic Classes
- (4) Husemoller: Fibre Bundles
- (5) Gilkey: Invariance Theory, The Heat Equation, and the Atiyah-Singer Index Theorem

Textbook: Notes by the instructor based on a book in progress.

Prerequisites: MATH 8061-8062. Differential Geometry and Topology I, II.

**MATH 9300 Seminar in Probability: Stochastic Calculus with Applications in
Finance
TR 9:30-10:50
Prof. A. Yilmaz**

Goals:

- (1) To introduce stochastic calculus. The central concept is the Itô integral which is a stochastic generalization of the Riemann-Stieltjes integral (where the integrands and the integrators are now stochastic processes such as Brownian motion). The stochastic calculus counterpart of the chain rule from ordinary calculus is called Itô's lemma.
- (2) To familiarize students with the fundamentals of quantitative finance which is essentially about the management of risk in a quantifiable manner. The central result is the Black-Scholes formula (1973 Nobel Prize in Economics) which provides a fair price for a risk-hedging security, specifically a European call option (i.e., the right to buy one share of a given stock at a specified price and time).

Book: "Stochastic Calculus for Finance II: Continuous-Time Models" by Steven E. Shreve (Springer).

Topics:

- (1) General probability theory (infinite probability spaces, random variables and distributions, expectations as Lebesgue integrals, change of measure)
- (2) Information and conditioning (σ -algebras, independence, conditional expectations)
- (3) Brownian motion (scaled random walks, definition and properties of Brownian motion, quadratic variation, Markov property)
- (4) Stochastic calculus (the Itô integral, Itô's lemma, Black-Scholes equation)
- (5) Risk-neutral pricing (Risk-neutral measure, Girsanov's theorem, Black-Scholes formula, martingale representation theorem, fundamental theorems of asset pricing)
- (6) Connections with partial differential equations (stochastic differential equations, Kolmogorov forward and backward equations, Feynman-Kac theorems, interest rate models)

Homework: Weekly assignments.

Style: The course will try to strike a balance between theory and applications. It will emphasize the motivation behind concepts, precise statements of results, and intuitive explanations. Technical proofs will be only sketched, but they will be augmented with informal arguments. Some homework assignments will extend the theory and others will be drawn from practical problems in quantitative finance.

Prerequisites: Undergraduate-level (calculus-based) probability. Previous exposure to graduate-level real analysis or probability theory is an advantage, but it is not necessary as the course will be self-contained in those regards.