Suppose that you roll a spherical ball along a plane curve without slipping, to trace out a curve on the sphere. Under what circumstances is the spherical curve periodic? [Differential-geometric problems of this general type have a long history, dating back to the classical era of Gaston Darboux (1842-1917)]. Specifically, if the plane curve is described by a prescribed curvature function $k(s)$, periodic in $s$ with period $L$, for what values of the radius of the sphere does the associated spherical curve have the property of being also periodic, with the same period $L$? We show how this problem can be transferred from the realm of ordinary differential equations to the realm of partial differential equations, where it becomes a straightforward question concerning the eigen-modes of an inhomogeneous vibrating string. Existence results and an asymptotic description of the eigen-values of the wave equation prove that yes, you can hear the shape of the periodic curves. Computer graphics will be used to illustrate some of the spherical curves that can occur.
Monday, April 11, 2005
Lecture at 4:00 PM (#),
refreshments at 3:00 PM. Room 617, Wachman Building
Department of Mathematics