Abstract. In this paper, we develop a family of second-order semi-implicit time integration methods for systems of ordinary differential equations (ODEs) with stiff damping term. The important feature of the new methods resides in the fact that they are capable of exactly preserving the steady states as well as maintaining the sign of the computed solution under the time step restriction determined by the nonstiff part of the system only. The new semi-implicit methods are based on the modification of explicit strong stability preserving Runge-Kutta (SSP-RK) methods and are proven to have a formal second order of accuracy, $A(\alpha)$-stability, and stiff decay. We illustrate the performance of the proposed SSP-RK-based semi-implicit methods on both scalar ODE examples and a system of ODEs arising from the semi-discretization of the shallow water equations with stiff friction term. The obtained numerical results clearly demonstrate that the ability of the introduced ODE solver to exactly preserve equilibria plays an important role in achieving high resolution when a coarse grid is used.