Algebraic Multigrid: theory and practice

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Abstract. This talk focuses on developing a generalized bootstrap algebraic multigrid algorithm for solving linear sparse matrix equations. As a motivation of the proposed generalization, we consider an optimal form of classical algebraic multigrid interpolation that has as columns eigenvectors with small eigenvalues of the generalized eigen-problem involving the system matrix and its symmetrized smoother. We use this optimal form to design an algorithm for choosing and analyzing the suitability of the coarse grid. In addition, it provides insights into the design of the bootstrap algebraic multigrid setup algorithm that we propose, which uses as a main tool a multilevel eigensolver to compute approximations to these eigenvectors. A notable feature of the approach is that it allows for general block smoothers and, as such, is well suited for systems of partial differential equations. In addition, we combine the GAMG setup algorithm with a least-angle regression coarsening scheme that uses local regression to improve the choice of the coarse variables. These new algorithms and their performance are illustrated numerically for scalar diffusion problems with highly varying (discontinuous) diffusion coefficient, Maxwell equations and for the linear elasticity system of partial differential equations.