TEMPLE UNIVERSITY
Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Room 617 Wachman Hall

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Stability of smooth extremals for variational integrals

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We consider the problem of minimizing

$$E(\boldsymbol{y}) = \int_\Omega W(\boldsymbol{x}, \nabla\boldsymbol{y}(\boldsymbol{x}))d\boldsymbol{x} \tag{1}$$

over vector fields $\boldsymbol{y}$ with given boundary conditions. We assume that W is sufficiently smooth and has a polynomial growth. Given a C^1 solution $\boldsymbol{y}$ of the Euler-Lagrange equation corresponding to (**??**), we study whether or not it is a local minimizer of E. We prove that if $\nabla\boldsymbol{y}(\boldsymbol{x})$ belongs to the region of uniform quasiconvexity of W for all $\boldsymbol{x} \in \Omega$, and if the second variation is uniformly positive then $\boldsymbol{y}$ is strong local minimizer of E. We use functional analysis to represent the increment of $E(\boldsymbol{y})$ corresponding to a general strong variation. The main device is a decomposition theorem that enables us to write a general variation as a sum of the weak and strong part. In the first lecture I will discuss the background and establish necessary conditions. In the second one I will prove the sufficiency result.