Abstract. Motivated by, among other applications, new types of signal processing problems and geometric integration in numerical analysis, there is an emerging interest in developing a constructive approximation theory for manifold-valued functions. The basic question is: Given a smooth manifold M, what is a constructive way to approximate an arbitrary smooth function $f: \mathbb{R} \to M$ with good accuracy? I shall describe a host of practical subdivision algorithms for interpolating or approximating manifold-valued data, focusing on the cases where the manifold is one of $S^n$ (the $n$-sphere), $SO(n)$, $SE(n)$, $SL(n)$ or a Stiefel manifold. Such algorithms are efficient to implement numerically (sometimes using the help of the SVD), produce an approximant in a multiresolution fashion, and enjoy certain desirable symmetry/invariance properties.

Despite such nice features, the mathematical analysis of such algorithms is at its infancy. I will present some recent results on the approximation order and smoothness properties of these nonlinear subdivision algorithms.